Kinematics

Position and Displacement

- **Position**, x, is a vector describing where an object is relative to an origin.
- **Displacement** is the change in position.

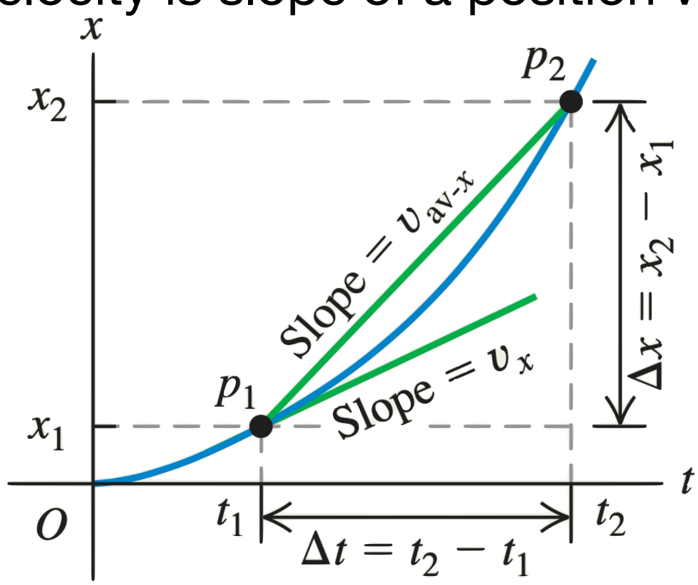
$$\Delta x = x_f - x_i$$

Velocity

- **Velocity** is the rate of change of position.

$$\boxed{v = \frac{dx}{dt}}$$

- Graphically, velocity is slope of a position vs. time graph.



- We can integrate to get the position function from a velocity function.

$$\boxed{x = x_0 + \int v dt}$$

- Graphically, change in position (i.e. displacement) is the area under a velocity vs. time graph.

- **Uniform motion** refers to motion at a constant velocity.

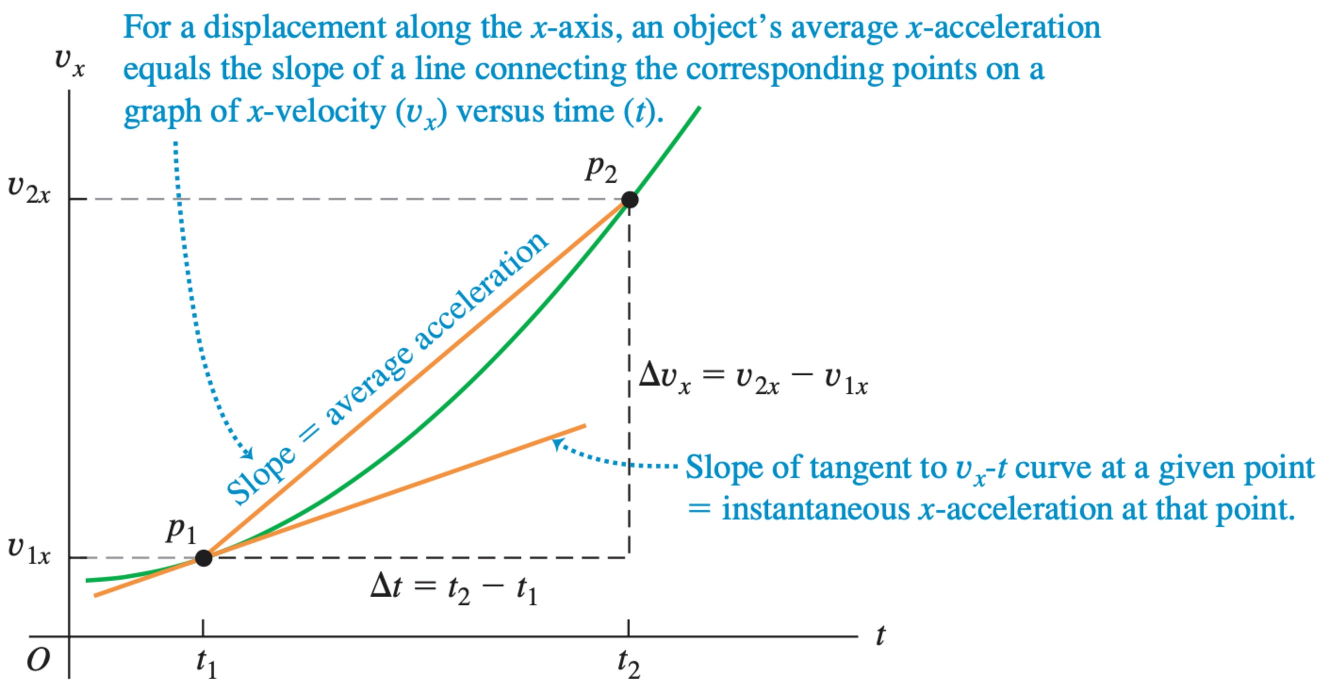
$$x = x_0 + vt$$

Acceleration

- **Acceleration** is the rate of change of velocity.

$$\boxed{a = \frac{dv}{dt} = \frac{d^2x}{dt^2}}$$

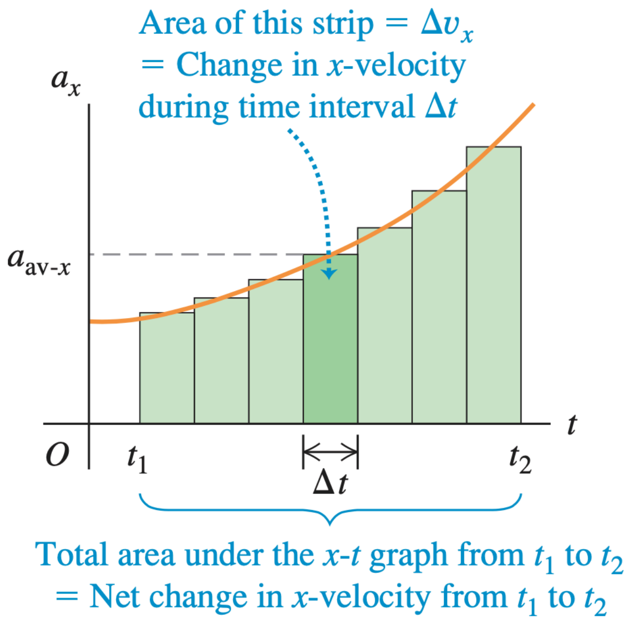
- Graphically, acceleration is the slope of a velocity vs. time graph.



- We can integrate to get the velocity function from an acceleration function.

$$\boxed{v = v_0 + \int a\,dt}$$

- Graphically, change in velocity is the area under an acceleration vs. time graph.



- The following equations are used for motion with constant acceleration.

$$\boxed{\begin{aligned} v &= v_0 + at \\ x &= x_0 + v_0 t + \frac{1}{2}at^2 \\ v^2 &= v_0^2 + 2a\left(x - x_0\right) \\ x - x_0 &= \left(\frac{v_0 + v}{2}\right)t \end{aligned}}$$

Example

A car starts from rest and has an acceleration $a(t) = 24t^2$, where a is in m/s^2 and t is in seconds.

a) How far does the car travel in 3 seconds?

b) When does the car have a velocity of 27 m/s?

Example

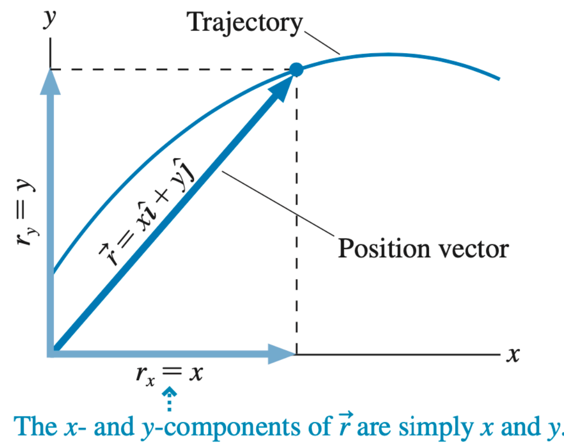
The position of a particle is given by $x(t) = 12t^2 - 4t^3$, where x is in metres and t is in seconds.

a) Determine the velocity and acceleration as functions of time.
b) At what time does the particle reach its maximum positive position? What is the particle's position at this time?
c) What distance does the particle move in the first four seconds?
d) What is the particle's displacement in the first four seconds?
e) At what time does the particle reach its maximum positive velocity? What is the particle's velocity at this time?

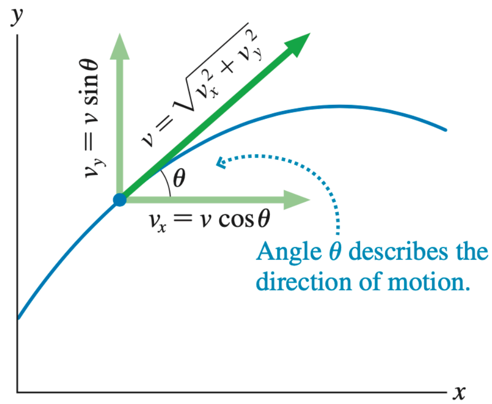
Motion in Two Dimensions

- The position of a particle moving in two dimensions can be written in terms of the *x*- and *y*-components.

$$\vec{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$



- As in one dimension, velocity and acceleration are the first and second derivatives of position with respect to time.
- As with position, the velocity and acceleration can be written in terms of the *x*- and *y*-components.



$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\mathbf{i}} + \frac{dv_y}{dt}\hat{\mathbf{j}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$$

Projectile Motion

- A **projectile** is an object that moves in two dimensions under the influence of only gravity.
- In the absence of air friction, $a_x = 0$ and $a_y = -g$.
- In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.
  - Horizontally, the motion is uniform with $v_x = v_0 \cos\theta$.

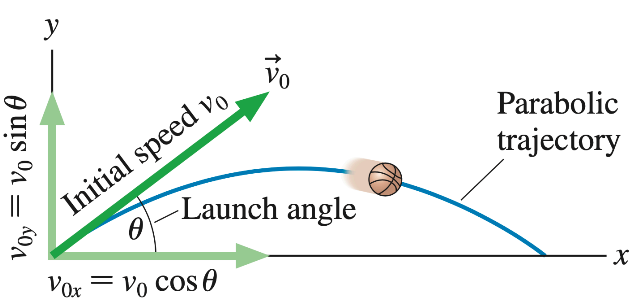
$$x = x_0 + v_x t$$

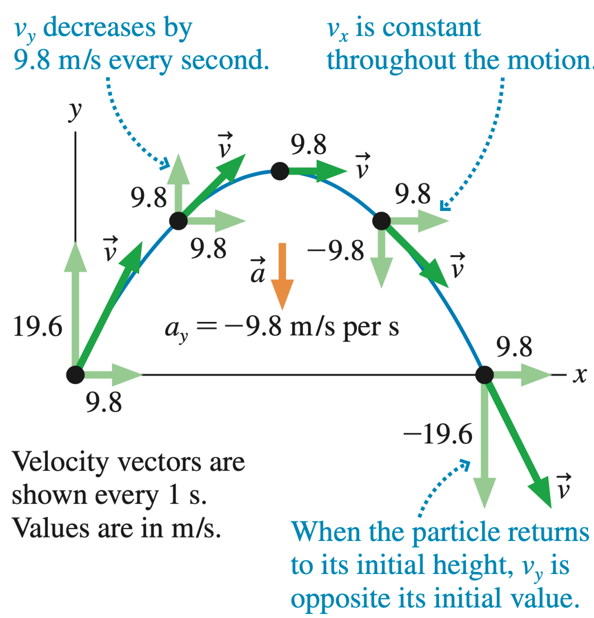
  - Vertically, the projectile experiences uniformly accelerated motion with $v_{0y} = v_0 \sin\theta$ and $a_y = -g$.

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

$$v_y = v_{0y} - g t$$

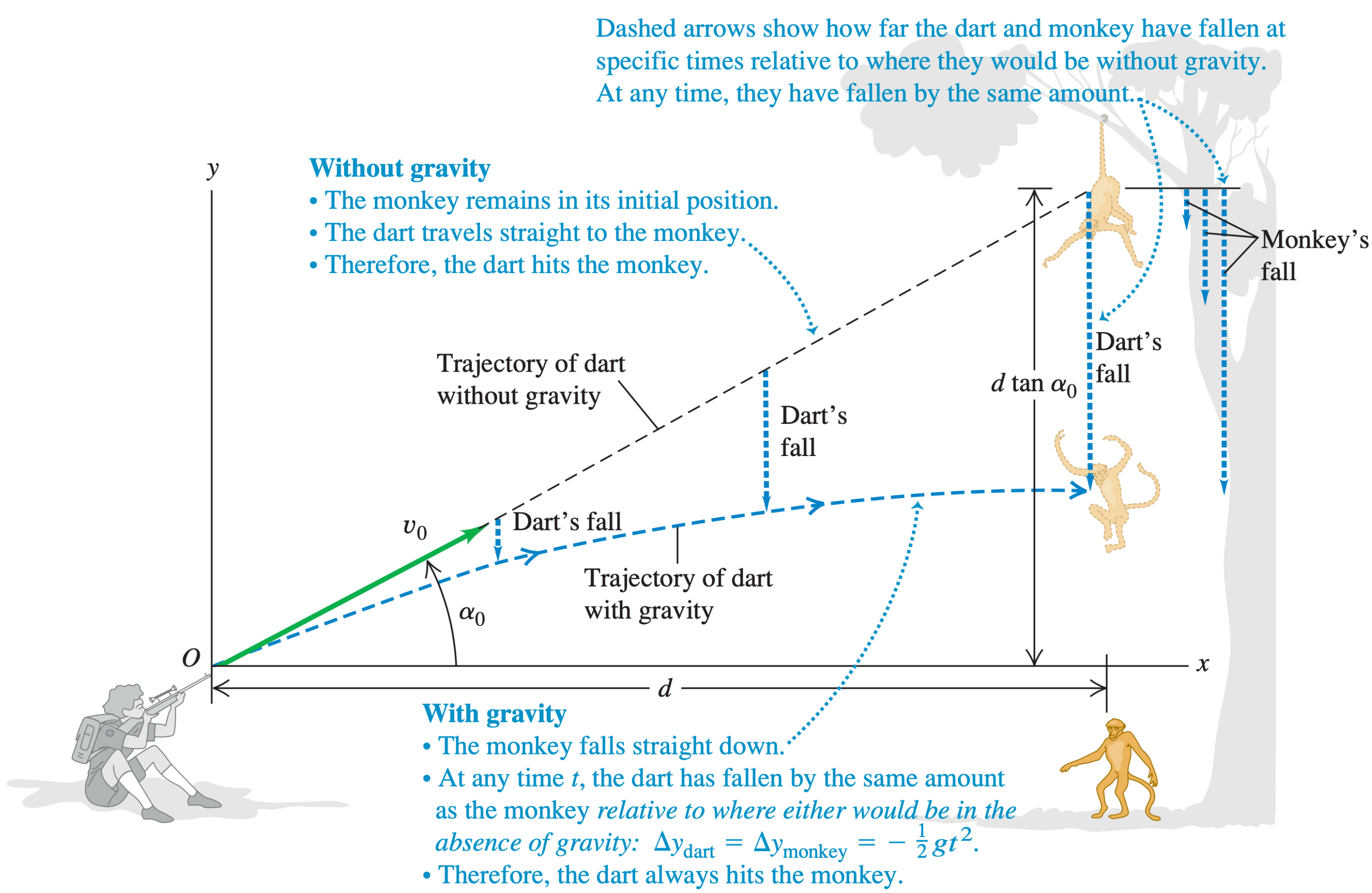
$$v_y^2 = v_{0y}^2 - 2g\left(y - y_0\right)$$





Example

A monkey escapes from the zoo and climbs a tree. After failing to entice the monkey down, the zookeeper fires a tranquilizer dart directly at the monkey. The monkey lets go at the instant the dart leaves the gun. Show that the dart will always hit the monkey, provided that the dart reaches the monkey before he hits the ground and runs away.



Example

Consider a projectile launched at an angle θ above level ground. Show that the range is maximized when the launch angle is 45°.

Example

The position of a particle moving in an *xy*-plane is given by $\vec{r} = \left(2t^3 - 5t\right)\hat{\mathbf{i}} + \left(6 - 7t^4\right)\hat{\mathbf{j}}$, with $\vec{r}$ in meters and *t* in seconds.

a) Determine the position, velocity and acceleration of the particle at $t = 2$ s. Express your answers in unit vector notation.

b) Determine the magnitude and direction of the velocity at $t = 2$ s.