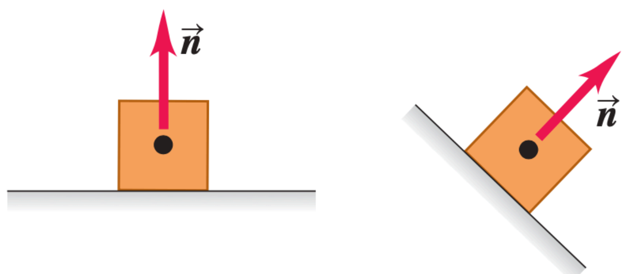
Dynamics

Forces

- **Gravitational force (weight)**: a long-range force exerted on a body by the pull of the earth.

$$\boxed{F_g = mg}$$

- **Normal force**: When an object rests or pushes on a surface, the surface exerts a push on it that is directed perpendicular to the surface.

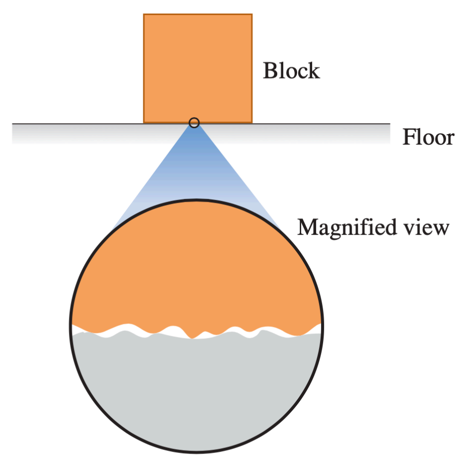


- **Friction force**: In addition to the normal force, a surface may exert a frictional force on an object, directed parallel to the surface, in the direction that opposes sliding.
    - If an object is at rest (**static**):

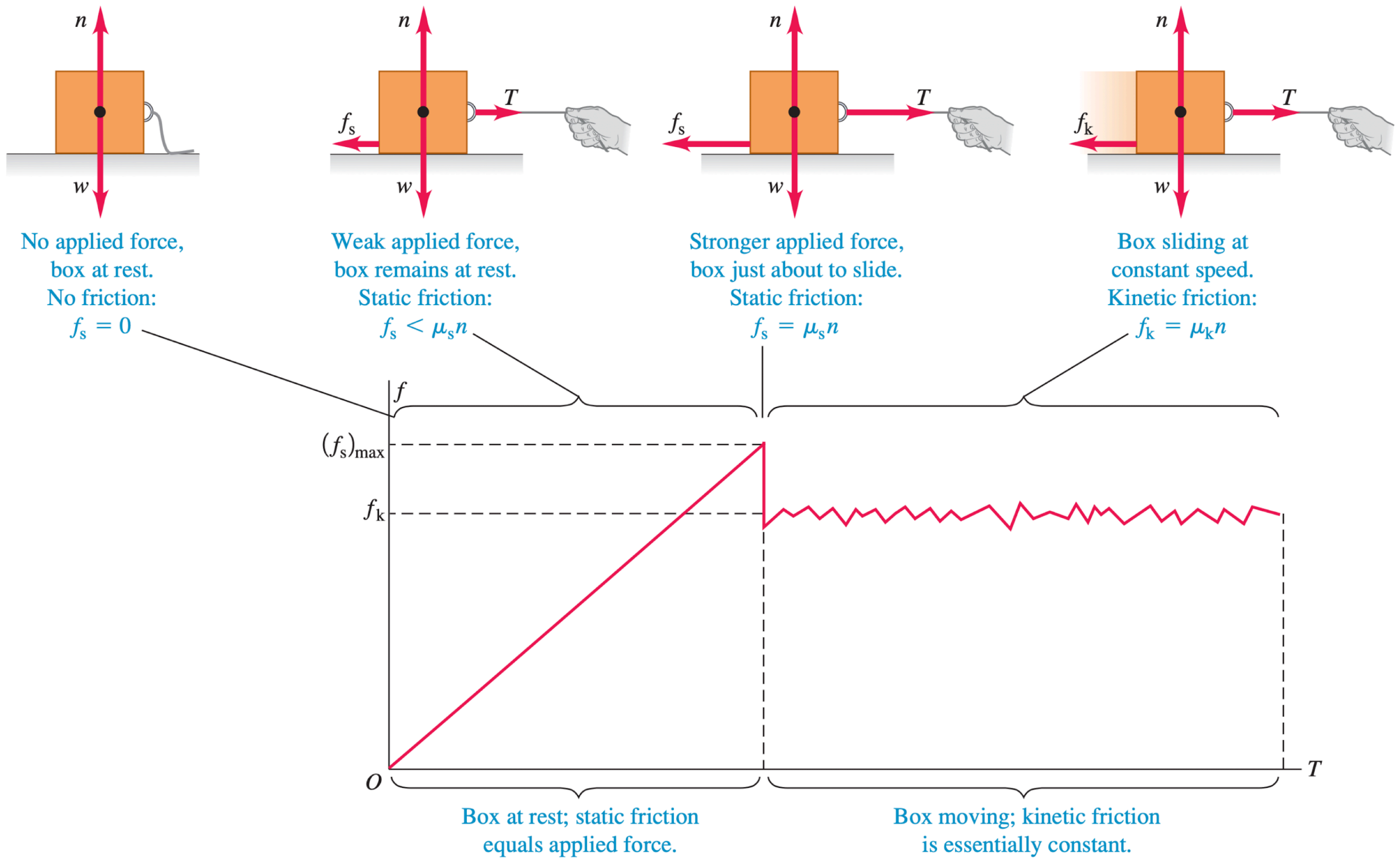
$$\boxed{F_f \leq \mu_s F_N}$$

    - If an object is in motion (**kinetic**):

$$\boxed{F_f = \mu_k F_N}$$

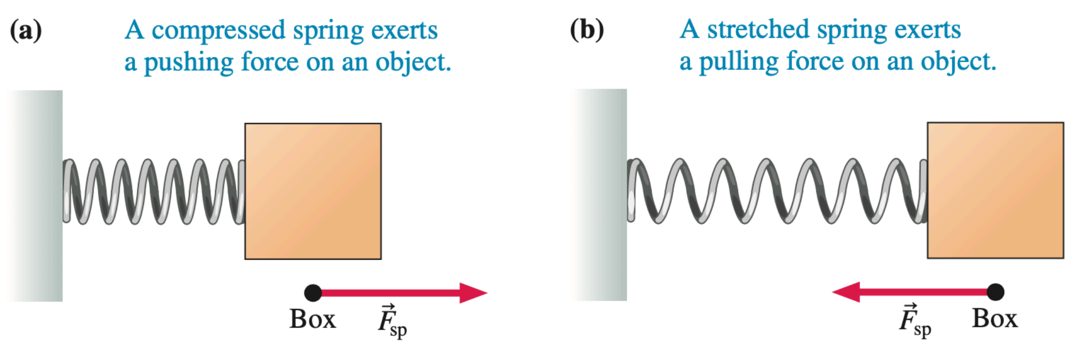


On a microscopic level, even smooth surfaces are rough; they tend to catch and cling.



- **Spring force**: An ideal stretched or compressed spring exerts a restoring force directly proportional to the spring's displacement from its equilibrium position (**Hooke's law**):

$$\boxed{F_s = kx}$$



Newton's Laws of Motion

- **Newton's First Law**: If all the forces acting on a body are balanced, then the object will not change speed or direction.

$$\boxed{\vec{F}_{\text{net}} = 0 \Leftrightarrow \vec{a} = 0}$$

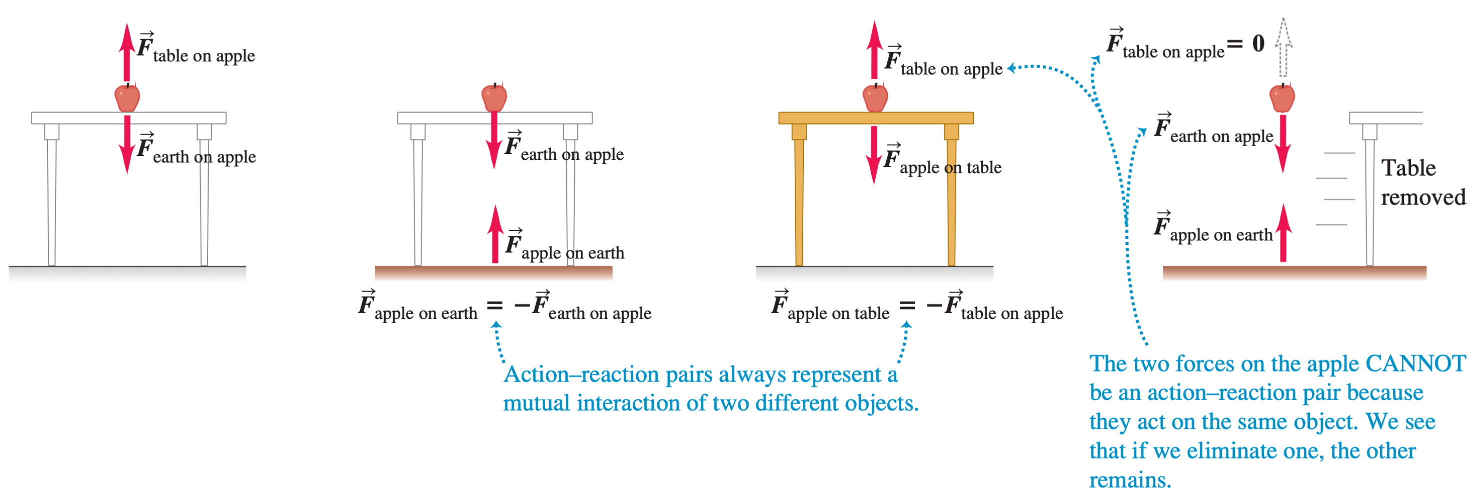
- **Newton's Second Law**: If there is an unbalanced force acting on an object, it will accelerate in the direction of the net force in inverse proportion to its mass.

$$\boxed{\vec{F}_{\text{net}} = m\vec{a}}$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

- **Newton's Third Law**: If body *A* exerts a force on body *B* (an "action"), then body *B* exerts an equal force back upon body *A* in the opposite direction (a "reaction").

$$\boxed{\vec{F}_{A\text{ on }B} = -\vec{F}_{B\text{ on }A}}$$



(a) The forces acting on the apple (b) The action–reaction pair for the interaction between the apple and the earth (c) The action–reaction pair for the interaction between the apple and the table (d) We eliminate one of the forces acting on the apple

Example

A 2 kg box is initially at rest on a horizontal surface. A force given by $F(t) = 6t^2$ is applied parallel to the surface where *F* is in Newtons and *t* is in seconds.

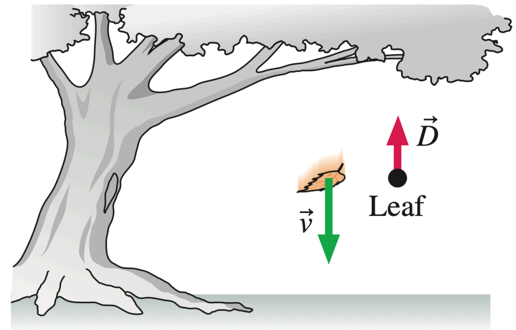
a) Determine the velocity of the box as a function of time if the surface is frictionless.

b) Determine the velocity of the box as as function of time if the coefficients of kinetic and static friction are 0.4 and 0.6 respectively.

Drag Force

- **Drag** is a resistive force of a fluid (e.g. air, water) on a moving object.
- Drag points opposite the direction of motion.
- **Terminal velocity** is the constant velocity attained when drag force is equal to the force of gravity.

Air resistance is a significant force on falling leaves. It points opposite the direction of motion.



- Drag increases in magnitude as the object's speed increases.
  - For small objects moving at very low speeds, the magnitude of the drag force is approximately proportional to the body's speed.

$$\boxed{F_D = bv}$$

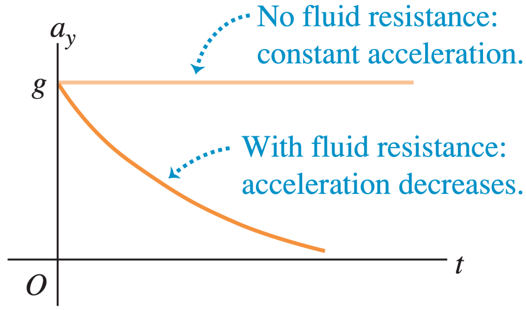
  - For other objects, the magnitude of the drag force is approximately proportional to the square of the speed.

$$\boxed{F_D = Cv^2}$$

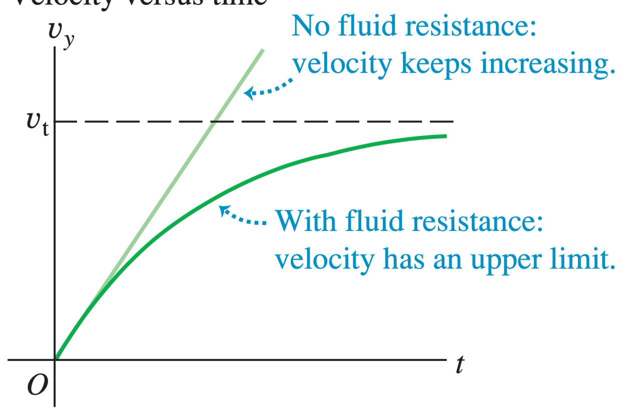
  - The drag coefficients *b* and *C* in the above equations depend on the size and shape of the body and the properties of the fluid.

- Apply Newton's second law and solve a differential equation to get velocity as a function of time.

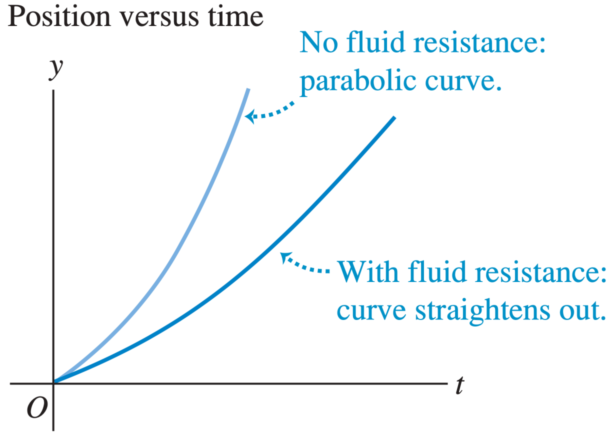
Acceleration versus time



Velocity versus time



Position versus time



Example

A metal ball of mass m is dropped at the surface of a bucket of oil and is allowed to fall to the bottom. The drag force is given by the equation $F_D = bv$.

a) What is the terminal velocity of the metal ball?
b) What is the velocity as a function of time?
c) What is the position as a function of time?