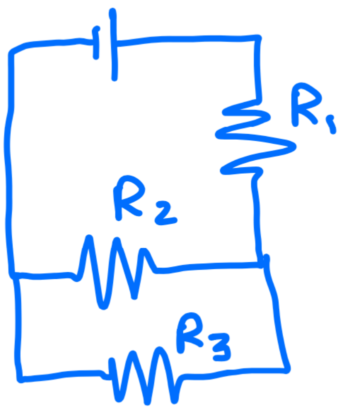
A heating coil has a resistance of 10 Ω and operates on 120 V. The coil is immersed in a container holding 2 L of water which has an initial temperature of 20 °C. The specific heat capacity of water is 4180 J/kgK.

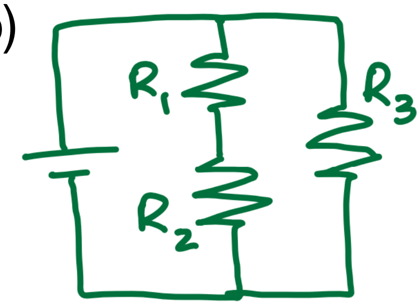
a) How long does it take the coil to bring the water to 100 °C?
b) At $0.11 per kWh, how much does it cost to heat the container of water?

Each of the following circuits is conducted of identical resistors. For each circuit, compare the power output of each resistor (e.g. $P_1 = 3P_2$).

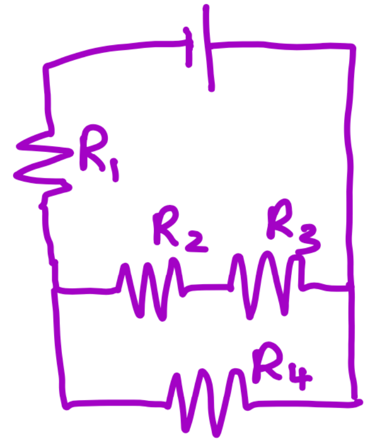
a)



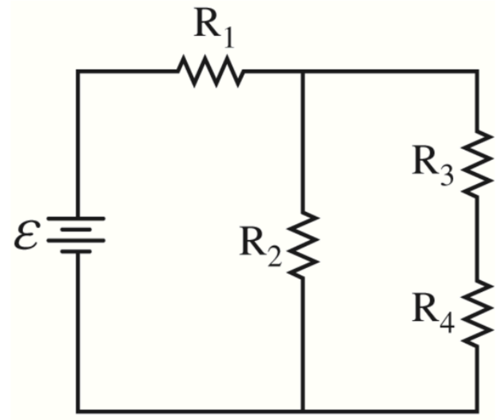
b)



c)



A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors, each of resistance R.

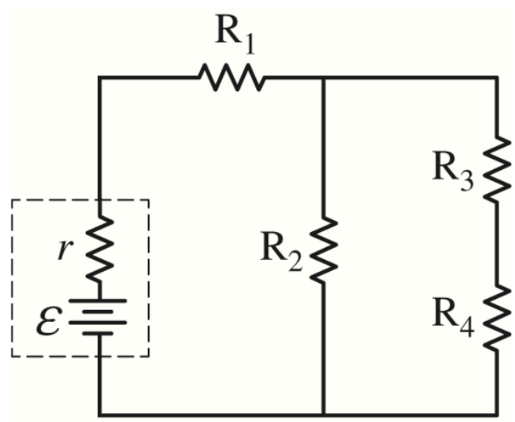


a) Derive an expression for the current I_1 in resistor R_1.

b) Derive an expression for the current I_3 in resistor R_3.

c) Derive an expression for the total power dissipated in the circuit.

When the ideal battery is connected in the original circuit, the total power dissipated in the circuit is $P_{original}$. The ideal battery is now replaced with a non ideal battery of emf $\mathcal{E}$ and internal resistance r. The total power dissipated in the new circuit is P_{new}.



d) Is P_{new} greater than, less than, or equal to $P_{original}$.

____ $P_{new} > P_{original}$ ____ $P_{new} < P_{original}$ ____ $P_{new} = P_{original}$

Explain.

A heating coil has a resistance of 10 Ω and operates on 120 V. The coil is immersed in a container holding 2 L of water which has an initial temperature of 20 °C. The specific heat capacity of water is 4180 J/kgK.

a) How long does it take the coil to bring the water to 100 °C?

$$P = \frac{V^2}{R} = \frac{(120)^2}{10} = 1440\text{ W}$$

$$Q = mc\Delta T$$
$$= (2)(4180)(100-20)$$
$$= 668\,800\text{ J}$$

$$P = \frac{W}{t} = \frac{Q}{t}$$

$$t = \frac{Q}{P} = \frac{668800}{1440}$$
$$= \boxed{464\text{ s}}$$

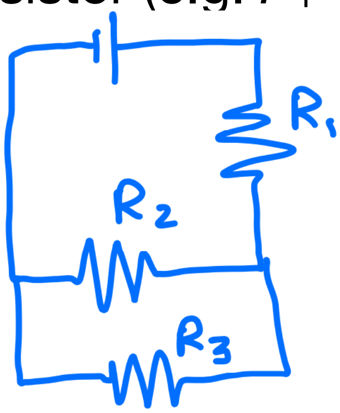
b) At $0.11 per kWh, how much does it cost to heat the container of water?

$$1\text{ kWh} = 1\frac{\text{kJ}}{\text{s}}\text{h} \times \frac{1000\text{ J}}{1\text{ kJ}} \times \frac{3600\text{ s}}{\text{h}} = 3.6\times10^6\text{ J}$$

$$668\,800\text{ J} \times \frac{1\text{ kWh}}{3.6\times10^6\text{ J}} \times \frac{\$0.11}{1\text{ kWh}} = \boxed{\$0.02}$$

Each of the following circuits is conducted of identical resistors. For each circuit, compare the power output of each resistor (e.g. $P_1 = 3P_2$).

a)



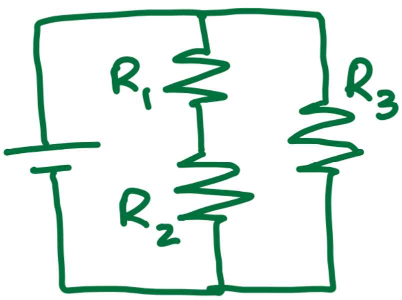
$I_2 = I_3 = \frac{1}{2} I_T$

$I_1 = I_T = 2I_2 = 2I_3$

$P = I^2 R$ so $P \propto I^2$

$\boxed{P_1 = 4P_2 = 4P_3}$

b)



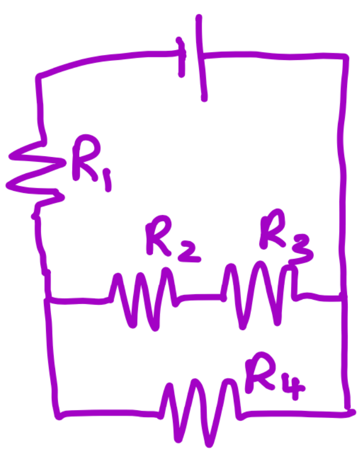
$V_1 = V_2 = \frac{1}{2} V_T$

$V_3 = V_T = 2V_1 = 2V_2$

$P = \frac{V^2}{R}$ so $P \propto V^2$

$\boxed{P_3 = 4P_1 = 4P_2}$

c)



$I_2 = I_3$

$I_4 = 2I_2 = 2I_3$

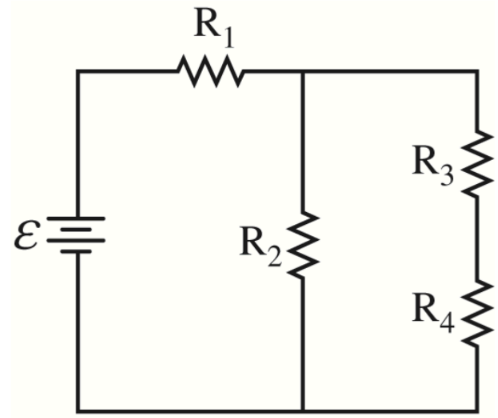
$I_1 = I_2 + I_4 = I_2 + 2I_2 = 3I_2$

$I_1 = 3I_2 = 3I_3 = \frac{3}{2} I_4$

$P = I^2 R$ so $P \propto I^2$

$\boxed{P_1 = \frac{9}{4} I_4 = 9I_2 = 9I_3}$

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors, each of resistance R.



a) Derive an expression for the current I_1 in resistor R_1.

$$R_{34} = R_3 + R_4 = R + R = 2R$$

$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_{34}} = \frac{1}{R} + \frac{1}{2R}$$

$$R_{234} = \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = \frac{2}{3}R$$

$$R_T = R_1 + R_{234} = R + \frac{2}{3}R = \frac{5}{3}R$$

$$I_1 = I_T = \frac{V_T}{R_T} = \frac{\mathcal{E}}{\frac{5}{3}R} = \boxed{\frac{3}{5}\frac{\mathcal{E}}{R}}$$

b) Derive an expression for the current I_3 in resistor R_3.

$$I_{234} = I_T = \frac{3}{5}\frac{\mathcal{E}}{R}$$

$$V_{234} = I_{234} R_{234} = \left(\frac{3}{5}\frac{\mathcal{E}}{R}\right)\left(\frac{2}{3}R\right) = \frac{2}{5}\mathcal{E}$$

$$V_{34} = V_{234} = \frac{2}{5}\mathcal{E}$$

$$V_3 = V_4 = \frac{1}{2}V_{34} = \frac{1}{2}\left(\frac{2}{5}\mathcal{E}\right) = \frac{1}{5}\mathcal{E}$$

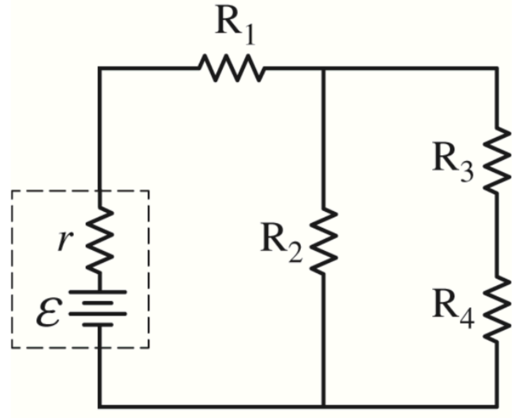
$$I_3 = \frac{V_3}{R_3} = \frac{\frac{1}{5}\mathcal{E}}{R} = \boxed{\frac{1}{5}\frac{\mathcal{E}}{R}}$$

c) Derive an expression for the total power dissipated in the circuit.

$$P_T = I_T V_T$$

$$= \left(\frac{3}{5}\frac{\mathcal{E}}{R}\right)\mathcal{E} = \boxed{\frac{3}{5}\frac{\mathcal{E}^2}{R}}$$

When the ideal battery is connected in the original circuit, the total power dissipated in the circuit is $P_{original}$. The ideal battery is now replaced with a nonideal battery of emf $\mathcal{E}$ and internal resistance r. The total power dissipated in the new circuit is P_{new}.



d) Is P_{new} greater than, less than, or equal to $P_{original}$.

___ $P_{new} > P_{original}$ _✓_ $P_{new} < P_{original}$ ___ $P_{new} = P_{original}$

Explain.

THE INTERNAL RESISTANCE OF THE NONIDEAL BATTERY RESULTS IN A INCREASED TOTAL RESISTANCE

$$R_{TOT,NEW} = R_{TOT,ORIGINAL} + r$$

WITH THE SAME EMF, TOTAL POWER IS INVERSELY PROPORTIONAL TO TOTAL RESISTANCE.

$$P_{TOT} = \frac{\mathcal{E}^2}{R_{TOT}}$$

WITH A GREATER TOTAL RESISTANCE, THERE IS A DECREASED TOTAL POWER.