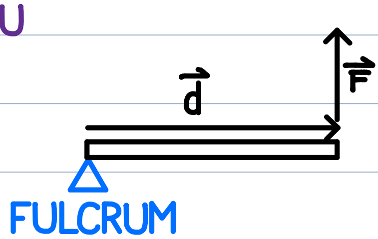
EQUILIBRIUM

TORQUE

- THE FULCRUM IS THE POINT OF SUPPORT AROUND WHICH A BODY ROTATES.
- TORQUE IS THE VECTOR PRODUCT BETWEEN FORCE AND DISPLACEMENT TO THE FULCRUM.

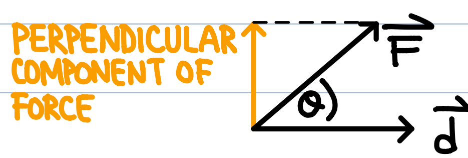
$$\boxed{\vec{\tau} = \vec{d} \times \vec{F}}$$

TAU



$\vec{\tau}$: TORQUE (N·m)
$\vec{d}$: DISPLACEMENT (m)
$\vec{F}$: FORCE (N)

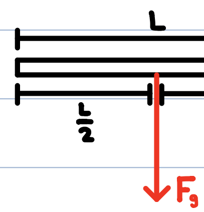
- VECTOR PRODUCT: AKA CROSS PRODUCT
  - TWO PERPENDICULAR VECTORS THAT MULTIPLY TO A VECTOR
  - IF NOT PERPENDICULAR, USE THE PERPENDICULAR COMPONENT.



$$\vec{\tau} = \vec{d} \times \vec{F}$$
$$= dF\sin\theta$$

CENTRE OF MASS

- FOR SIMPLE OBJECTS OF UNIFORM DENSITY, THE CENTRE OF MASS IS LOCATED AT THE OBJECT'S GEOMETRIC CENTRE.
- FOR THE PURPOSES OF CALCULATION, WE CAN TREAT AN ODDLY-SHAPED OBJECT AS IF AS IF ALL ITS MASS IS CONCENTRATED AT THIS SINGLE POINT.
- THIS IS WHERE THE FORCE OF GRAVITY ACTS ON AN OBJECT.



STATIC EQUILIBRIUM

- AN OBJECT IS IN STATIC EQUILIBRIUM IF IT NEITHER MOVES LINEARLY (TRANSLATIONAL EQUILIBRIUM) NOR TURNS (ROTATIONAL EQUILIBRIUM).

- CONDITIONS FOR STATIC EQUILIBRIUM:
  - NET FORCE IS ZERO.
  - NET TORQUE IS ZERO.

$$\boxed{\begin{array}{c}\vec{F}_{NET}=0 \\ \vec{\tau}_{NET}=0\end{array}}$$

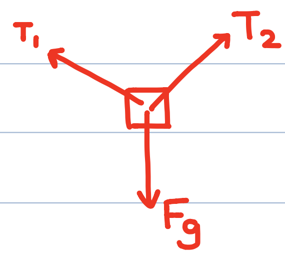
$\vec{F}_{NET}=0 \rightarrow \Sigma F_x=0$, $\Sigma F_y=0$

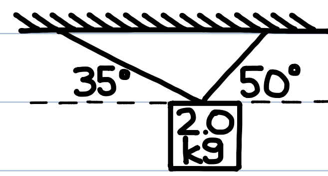
$\vec{\tau}_{NET}=0 \rightarrow \tau_{cw}=\tau_{ccw}$

- WHEN IN ROTATIONAL EQUILIBRIUM, THE FULCRUM CAN BE PLACED ANYWHERE. PLACE THE FULCRUM STRATEGICALLY TO ELIMINATE A TORQUE

EXAMPLE

A 2.0 kg SIGN IS SUSPEND BY TWO WIRES AS SHOWN. DETERMINE THE TENSION IN EACH WIRE.

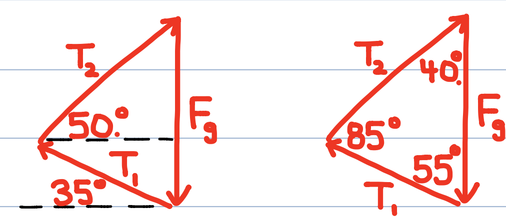




NOTE THAT ALL FORCES ACT ON A SINGLE POINT. WE DO NOT NEED TO CONSIDER ROTATIONAL EQUILIBRIUM.

$$\vec{F}_{NET} = 0$$

$$\vec{F}_g + \vec{T}_1 + \vec{T}_2 = 0 \longrightarrow$$



$$\frac{F_g}{\sin 85°} = \frac{T_1}{\sin 40°} = \frac{T_2}{\sin 55°}$$

$$T_1 = \frac{\sin 40°}{\sin 85°} F_g$$

$$= \frac{\sin 40°}{\sin 85°} mg$$

$$= \frac{\sin 40°}{\sin 85°}(2.0)(9.8)$$

$$= 12.65\text{ N} \rightarrow 13\text{ N}$$

$$T_2 = \frac{\sin 55°}{\sin 85°} F_g$$

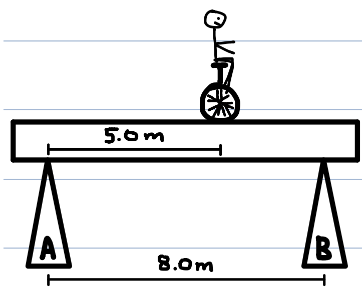
$$= \frac{\sin 55°}{\sin 85°} mg$$

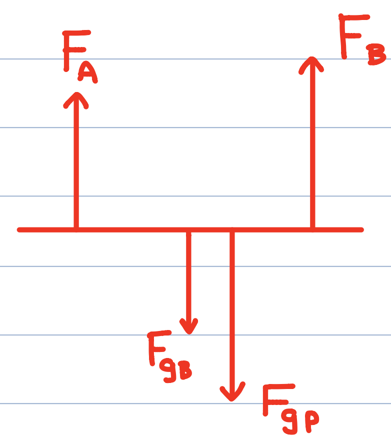
$$= \frac{\sin 55°}{\sin 85°}(2.0)(9.8)$$

$$= 16.12\text{ N} \rightarrow 16\text{ N}$$

EXAMPLE

A CIRCUS PERFORMER ON A UNICYCLE OF TOTAL MASS 55 kg RIDES ACROSS A UNIFORM 30 kg BEAM. THE SUPPORTS ARE PLACED AT EQUAL DISTANCES FROM THE ENDS OF THE BEAM. WHEN HE IS AT THE POSITION SHOWN, DETERMINE THE FORCES EXERTED BY THE SUPPORTS ON THE BEAM.





$$\tau_{cw} = \tau_{ccw}$$

$$F_A(8.0) = F_{gB}(4.0) + F_{gP}(3.0)$$

$$F_A(8.0) = m_B g(4.0) + m_P g(3.0)$$

$$F_A = \frac{m_B g(4.0) + m_P g(3.0)}{8.0}$$

$$= \frac{(30)(9.8)(4.0) + (55)(9.8)(3.0)}{8.0}$$

$$= 349\text{ N (UP)}$$

$$F_{NET} = 0$$

$$F_A + F_B - F_{gB} - F_{gP} = 0$$

$$F_B = F_{gB} + F_{gP} - F_A$$

$$= m_B g + m_P g - F_A$$

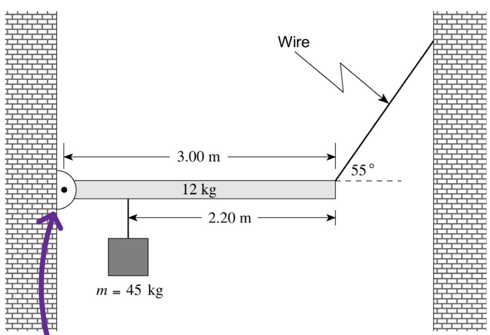
$$= (30)(9.8) + (55)(9.8) - 349$$

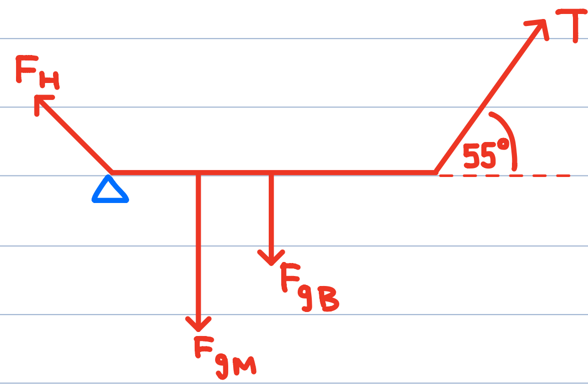
$$= 484\text{ N (UP)}$$

CAN ALSO BE SOLVED USING $\tau_{cw} = \tau_{ccw}$

EXAMPLE

A UNIFORM 12 kg BEAM OF LENGTH 3.00 m HOLDING A 45 kg MASS IS ATTACHED BY A WIRE TO A WALL AS SHOWN. WHAT IS THE TENSION IN THE WIRE?





IF ATTACHED TO A SURFACE (E.G. WITH A HINGE), THE SURFACE CAN EXERT A FORCE IN ANY DIRECTION.

$$\tau_{cw} = \tau_{ccw}$$

$$F_{gM}(0.80) + F_{gB}(1.50) = T(3.00)\sin 55°$$

$$m_M g(0.80) + m_B g(1.50) = T(3.00)\sin 55°$$

$$T = \frac{m_M g(0.80) + m_B g(1.50)}{3.00 \sin 55°}$$

$$= \frac{(45)(9.8)(0.80) + (12)(9.8)(1.50)}{3.00 \sin 55°}$$

$$= 215.34 \text{ N}$$

$$\longrightarrow 220 \text{ N}$$