

Fluids

Fluids

- **Fluids** are substances with no fixed shape.
- Gasses have space between molecules allowing them compress or expand.
- Liquids cannot be compressed.
- An **ideal fluid** is incompressible and has no viscosity (resistance to flow).

Density

- **Mass density** is mass per unit volume.

$$\rho = \frac{m}{V}$$

Densities of Some Common Substances

Material	Density (kg/m ³)	Material	Density (kg/m ³)
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerine	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

Example

The density of air at 20°C is 1.20 kg/m³. What is the approximate mass of air in this room?

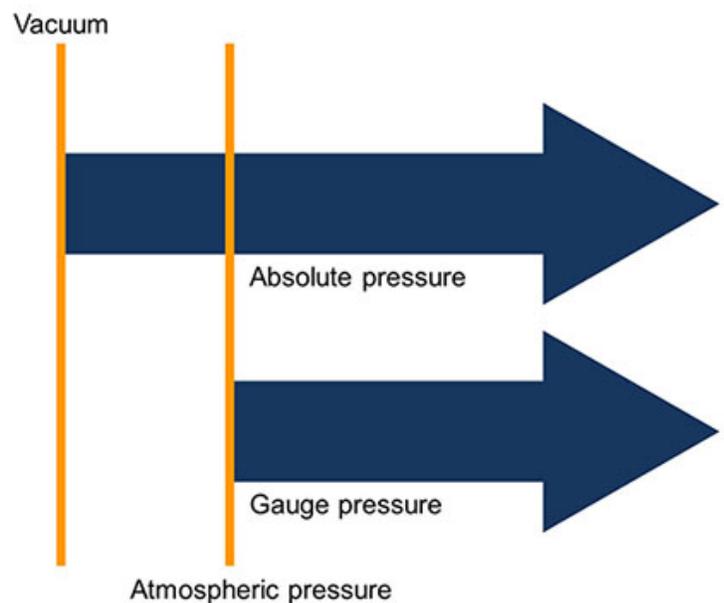
Pressure

- **Pressure** is the magnitude of the perpendicular force component exerted per unit area

$$P = \frac{F_{\perp}}{A}$$

- SI unit: Pascal (Pa)
 - equivalent to N/m^2
- Also commonly used is the standard atmosphere (atm).
 - 1 atm is Earth's atmospheric pressure at sea level.
 - $1 \text{ atm} = 1.01325 \times 10^5 \text{ Pa}$

- **Absolute pressure** is the total pressure measured relative to a perfect vacuum (zero pressure).
- **Gauge pressure** is the pressure measured with respect to the atmospheric pressure.
- The absolute pressure is the sum of a reference pressure P_0 , such as the atmospheric pressure P_{atm} , and the gauge pressure P_{gauge} .



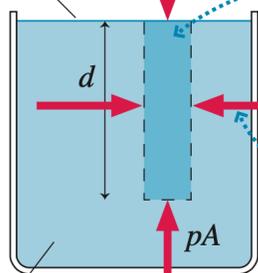
$$P = P_0 + P_{\text{gauge}}$$

Hydrostatics

- **Hydrostatics** is the study of fluids at **hydrostatic equilibrium** (i.e. not flowing).
- At equilibrium, the net force is zero.
- Pressure in a fluid is a result of the motion of the molecules that make up the fluid. The force exerted by the fluid is due to molecules colliding with their surroundings.
- The force resulting from fluid pressure at a particular point in a fluid has the same magnitude in all directions.

Whatever is above the liquid pushes down on the top of the cylinder.

Pressure p_0 at the surface

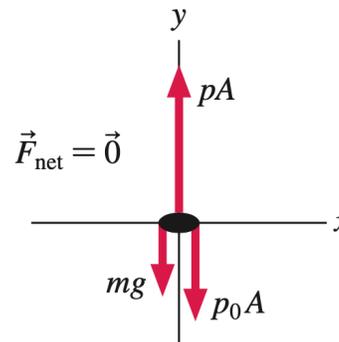


This cylinder of liquid (depth d , cross-section area A) is in static equilibrium.

The liquid on each side of the cylinder pushes in on the cylinder.

The liquid beneath the cylinder pushes up on the cylinder. The pressure at depth d is p .

Liquid of density ρ



Free-body diagram of the column of liquid. The horizontal forces cancel and are not shown.

- Consider a cylindrical column of liquid in equilibrium ($F_{\text{net}} = 0$).

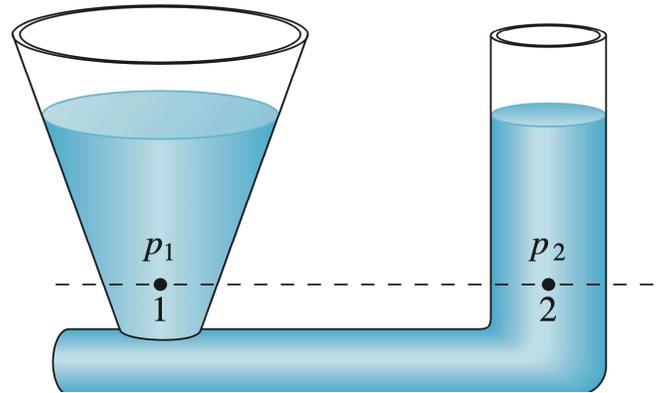
$$\begin{aligned}
 P_{\text{bottom}}A &= P_0A + mg \\
 P_{\text{bottom}} &= P_0 + \frac{mg}{A} \\
 &= P_0 + \frac{\rho Vg}{A} \\
 &= P_0 + \rho gh
 \end{aligned}$$

- A fluid in equilibrium exerts pressure due to its weight. The gauge pressure in a fluid of uniform density is given by

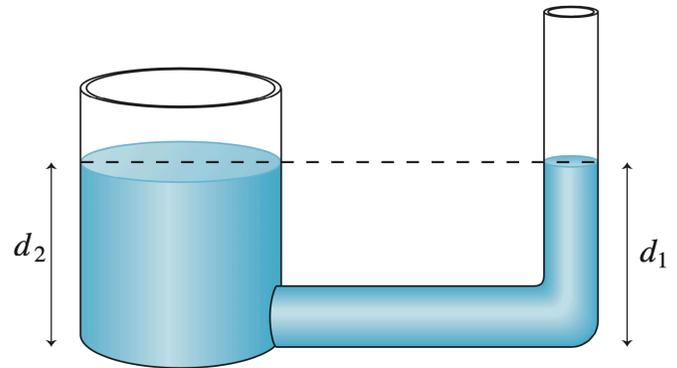
$$P_{\text{gauge}} = \rho gh$$

where h is the depth below the fluid's surface.

- In hydrostatic equilibrium, the pressure is the same at all points on a horizontal line through a connected liquid of a single kind.



- A connected liquid in hydrostatic equilibrium rises to the same height in all open regions of the container.



Example

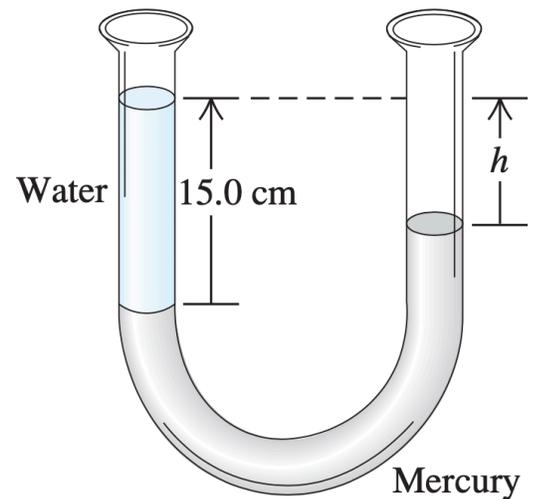
A scuba diver descends to a depth of 40 m.

- What pressure is read by her scuba equipment?
- What is the absolute pressure at this point?

Example

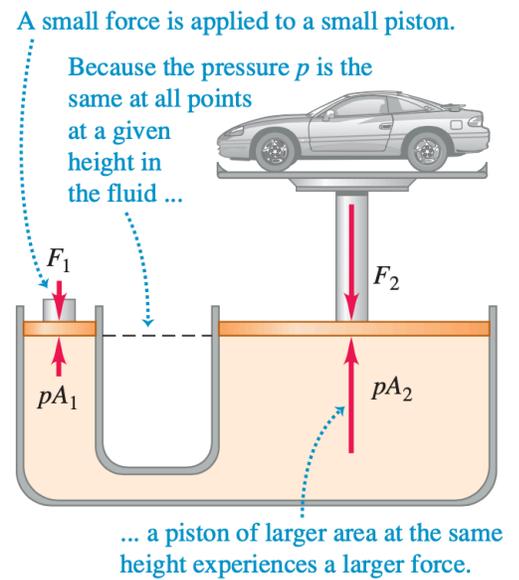
A U-shaped tube open to the air at both ends contains some mercury (density $1.36 \times 10^4 \text{ kg/m}^3$). A quantity of water is carefully poured into the left arm of the U-shaped tube until the vertical height of the water column is 15.0 cm.

- What is the gauge pressure at the water–mercury interface?
- Calculate the vertical distance h from the top of the mercury in the right-hand arm of the tube to the top of the water in the left-hand arm.



- **Pascal's Principle:** When force is applied to an enclosed incompressible fluid, the change in pressure is transmitted equally to all parts of the fluid and to the walls of its container.

$$P_1 = P_2$$

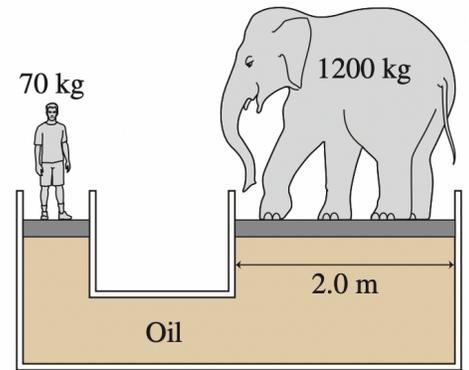


- With a hydraulic lift, a given force applied over a given distance can be transformed to a greater force applied over a smaller distance.

Example

A 70 kg student balances a 1200 kg elephant on a hydraulic lift which is filled with oil of density 900 kg/m^3 .

- What is the diameter of the piston the student is standing on?
- When a second student joins the first, the piston sinks 35 cm. What is the second student's mass?

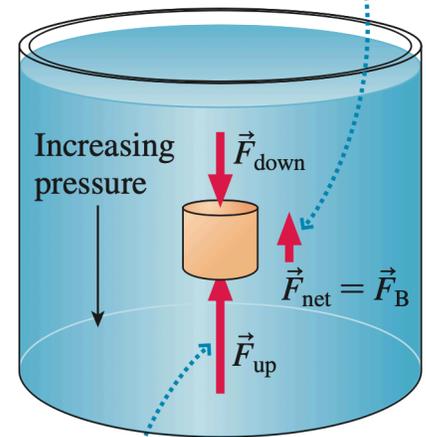


Buoyancy

- The **buoyant force** is a net upward force exerted on an object by a fluid.
- The buoyant force is a result of the pressure difference at the top and bottom of the object in a fluid.

$$\begin{aligned}F_b &= F_{\text{bottom}} - F_{\text{top}} \\ &= P_{\text{bottom}}A - P_{\text{top}}A \\ &= \rho g h_{\text{bottom}}A - \rho g h_{\text{top}}A \\ &= \rho g (h_{\text{bottom}} - h_{\text{top}})A \\ &= \rho Vg\end{aligned}$$

The net force of the fluid on the cylinder is the buoyant force \vec{F}_B .



$F_{\text{up}} > F_{\text{down}}$ because the pressure is greater at the bottom. Hence the fluid exerts a net upward force.

- **Archimedes' Principle:** The buoyant force exerted on an object by a fluid is directed upwards and has a magnitude equal to the weight of the fluid displaced by the object.

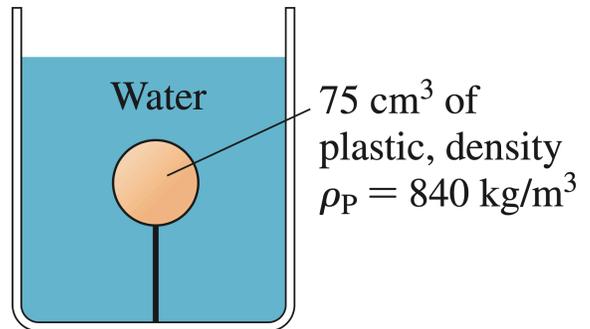
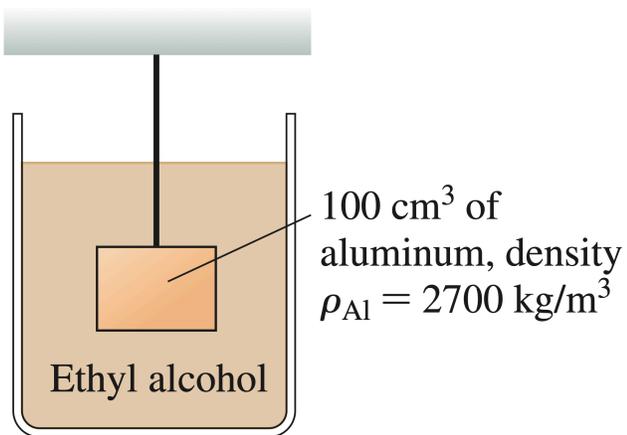
$$F_b = \rho Vg$$

where ρ is the fluid density and V is the volume of the displaced fluid.

- For a fully-submerged object, the displaced volume is equal to the total volume of the object.
- For a partially-submerged object, the displaced volume is equal to the volume of the portion of the object which is below the surface of the fluid.
- Objects which are more dense than a fluid will sink.
- Objects which are less dense than a fluid will float.

Example

Determine the tension in each string. The density of ethyl alcohol is 789 kg/m^3 .



Example

What fraction of the volume of an iceberg (density 917 kg/m^3) would be visible if the iceberg floats in the ocean (salt water, density 1024 kg/m^3)?

Hydrodynamics

- **Hydrodynamics** is the study of fluids in motion.
- Three assumptions for an ideal fluid:
 1. Fluid is incompressible.
 2. Fluid is non-viscous.
 3. Fluid velocity at any fixed point in the fluid is constant (**laminar flow**).
- **Flow rate** is the volume of a fluid that passes a particular point in a given time.

$$Q = \frac{V}{t} = \frac{Ad}{t} = Av$$

- **Equation of Continuity:** The flow rate of an ideal fluid is constant at all points in a tube.
- The mass of a moving fluid does not change as it flows. As mass is conserved:

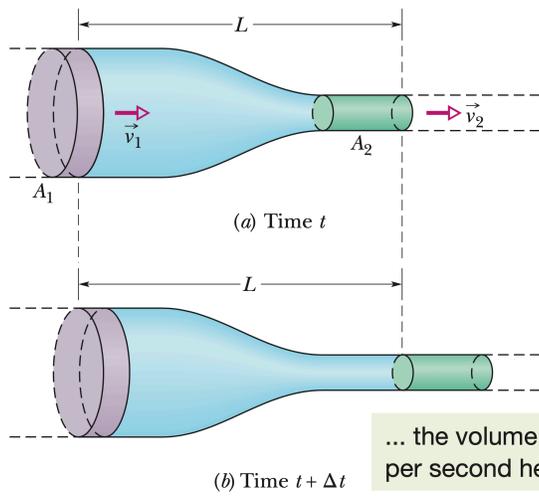
$$\frac{m_1}{t} = \frac{m_2}{t}$$
$$\frac{\rho_1 V_1}{t} = \frac{\rho_2 V_2}{t}$$
$$\frac{\rho_1 A_1 d_1}{t} = \frac{\rho_2 A_2 d_2}{t}$$
$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$A_1 v_1 = A_2 v_2$$

If fluid is incompressible,
 $\rho_1 = \rho_2$.

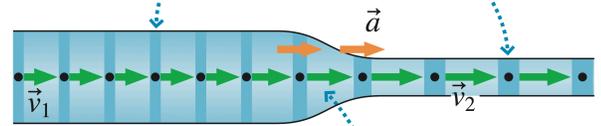
- Flow rate is greater at the narrower parts of a tube.

The volume flow per second here must match ...



... the volume flow per second here.

A fluid element coasts at steady speed through the constant-diameter segments of the tube.



As a fluid element flows through the tapered section, it speeds up. Because it is accelerating, there must be a force acting on it.

Example

A shower head has 20 circular openings, each with radius 1 mm. The shower head is connected to a pipe with radius 8 mm. If the speed of water in the pipe is 3 m/s, what is its speed as it exits the shower-head openings?

- **Bernoulli's Equation** relates the pressure, flow speed and elevation for any two points in an ideal fluid.
- Applying the work-energy theorem:

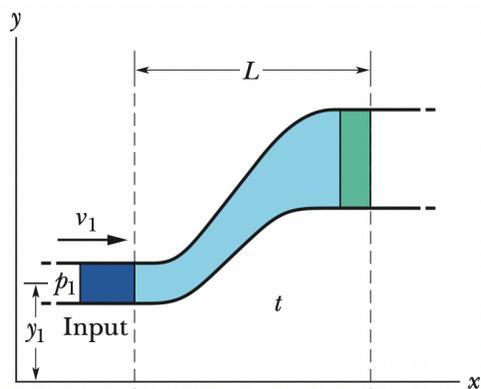
$$W_1 + E_{k,1} + E_{p,1} = W_2 + E_{k,2} + E_{p,2}$$

$$F_1 d_1 + \frac{1}{2} m v_1^2 + m g y_1 = F_2 d_2 + \frac{1}{2} m v_2^2 + m g y_2$$

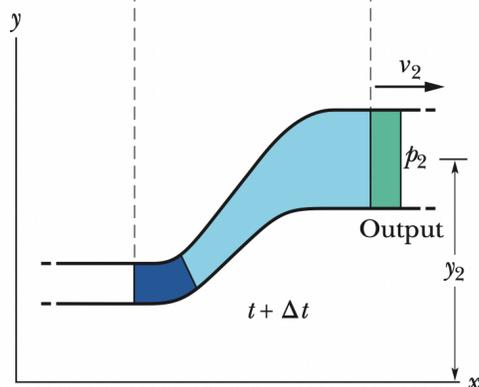
$$\frac{F_1 d_1}{V} + \frac{1}{2} \frac{m v_1^2}{V} + \frac{m g y_1}{V} = \frac{F_2 d_2}{V} + \frac{1}{2} \frac{m v_2^2}{V} + \frac{m g y_2}{V}$$

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

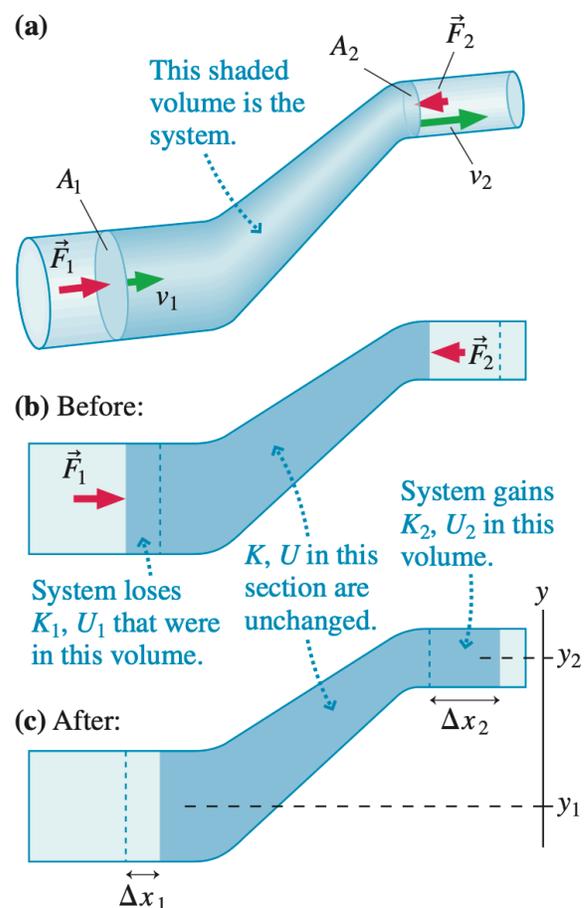
where y represents elevation (as opposed to depth h , as seen in the hydrostatic pressure equation).



(a)



(b)



(a)

(b) Before:

(c) After:

- Special cases:

1. Fluid at rest or moving at a constant velocity ($v_1 = v_2$):

$$P_1 + \rho g y_1 = P_2 + \rho g y_2$$

$$\Delta P = \rho g (y_1 - y_2) = \rho g \Delta h$$

- For a fluid at equilibrium, the pressure increases with depth. This agrees with the hydrostatic pressure equation.

2. Horizontal Pipe (constant elevation, $y_1 = y_2$):

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

- In a horizontal pipe, the faster a fluid flows, the lower the pressure will be.

3. No change in pressure (e.g. hole in a leaky container with an open top):

$$\frac{1}{2} \rho v_1^2 + \rho g y_1 = \frac{1}{2} \rho v_2^2 + \rho g y_2$$

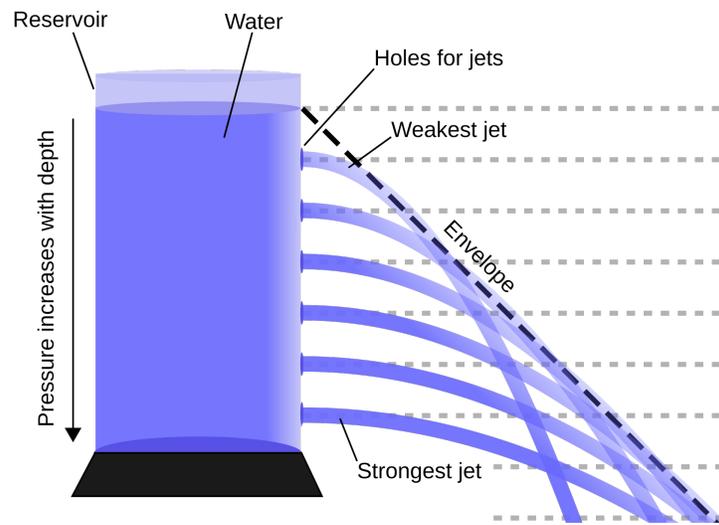
$$\frac{1}{2} v_1^2 + g y_1 = \frac{1}{2} v_2^2 + g y_2$$

$$v_2^2 = v_1^2 + 2g (y_1 - y_2)$$

$$v_2 = \sqrt{2gh}$$

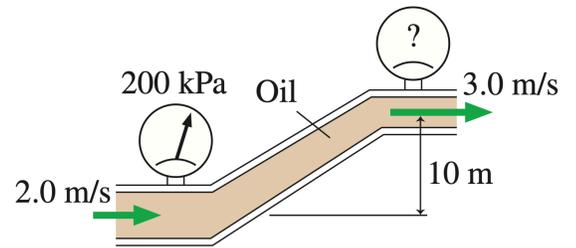
By the equation of continuity, if $A_2 \ll A_1$, $v_1 \approx 0$.

- **Torricelli's theorem:** The speed of an ideal fluid through a hole in the wall of the tank filled to a height h above the hole is the same as the speed that a body would acquire in falling freely from a height h .



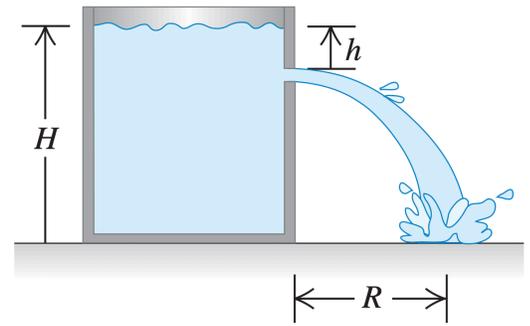
Example

Oil of density 900 kg/m^3 flows through a pipe. What does the top pressure gauge read?



Example

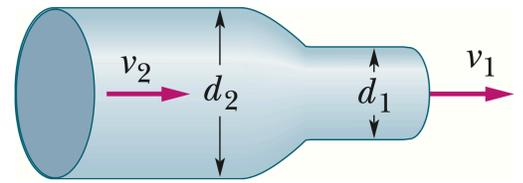
Water stands at a depth $H = 40$ cm in a large, open tank whose side walls are vertical. A hole is made in one of the walls at a depth $h = 10$ cm below the water surface.



- At what distance R does the stream strike the floor?
- At what depth should a second hole be made to give the same value of R ?

Example

Water flows through a horizontal pipe and then out into the atmosphere at a speed $v_1 = 15 \text{ m/s}$. The diameters of the left and right sections of the pipe are 5 cm and 3 cm.

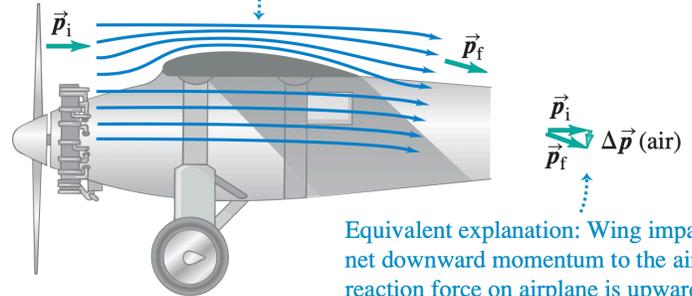


- What volume of water flows into the atmosphere during a 10 min period?
- What is the speed v_2 in the left section of the pipe?
- What is the gauge pressure in the left section of the pipe?

Example

An airplane wing has a surface area of 20 m^2 . The speed of airflow over the top surface is 70 m/s , and the speed of airflow under the bottom surface is 50 m/s . The density of air is 1.225 kg/m^3 . Determine the net lift force acting on the wing.

Flow lines are crowded together above the wing, so flow speed is higher there and pressure is lower.



Equivalent explanation: Wing imparts a net downward momentum to the air, so reaction force on airplane is upward.