Physics 12
M. Lam

Graphical Analysis II - Solutions

Note: $k_{experimental}$ will vary for hand-drawn graphs and best fit lines.

1. $v \propto \sqrt{h};\ v = k\sqrt{h}$

   Velocity is directly proportional to the square root of height.

   $$k_{\text{experimental}} = \frac{\Delta v}{\Delta\sqrt{h}} = 4.31\frac{\text{m}^{\frac{1}{2}}}{\text{s}}$$

   Using kinematics or the conservation of energy:

   $$k_{\text{theoretical}} = \sqrt{2g} = 4.4\frac{\text{m}^{\frac{1}{2}}}{\text{s}}$$

2. $d \propto t^2;\ d = kt^2$

   Displacement is directly proportional to the square of time.

   $$k_{\text{experimental}} = \frac{\Delta d}{\Delta t^2} = 4.77\frac{\text{m}}{\text{s}^2}$$

   Using kinematics:

   $$k_{\text{theoretical}} = \frac{1}{2}g = 4.9\frac{\text{m}}{\text{s}^2}$$

3. $a \propto \frac{1}{m};\ a = \frac{k}{m}$

   Acceleration is inversely proportional to mass.

   $$k_{\text{experimental}} = \frac{\Delta a}{\Delta \frac{1}{m}} = 1.94\ \text{N}$$

   Using Newton's second law:

   $$k_{\text{theoretical}} = F = 2.0\ \text{N}$$

4. $v \propto t;\ v = kt$

Velocity is directly proportional to time.

$$k_{\text{experimental}} = \frac{\Delta v}{\Delta t} = 0.397 \frac{\text{m}}{\text{s}^2}$$

Using impulse:

$$k_{\text{theoretical}} = \frac{F}{m} = a = 0.40 \frac{\text{m}}{\text{s}^2}$$

5. $t \propto \frac{1}{P};\ t = \frac{k}{P}$

Time is inversely proportional to power.

$$k_{\text{experimental}} = \frac{\Delta t}{\Delta \frac{1}{P}} = 203\ \text{kJ} = 2.03 \times 10^5\ \text{J}$$

Using power and kinetic energy:

$$k_{\text{theoretical}} = \frac{1}{2} m v^2 = 200\ \text{kJ} = 2.0 \times 10^5\ \text{J}$$

6. $h \propto v_i^2;\ h = k v_i^2$

Height is directly proportional to the square of initial speed.

$$k_{\text{experimental}} = \frac{\Delta h}{\Delta v_i^2} = 0.0501 \frac{\text{s}^2}{\text{m}}$$

Using the conservation of energy:

$$k_{\text{theoretical}} = \frac{1}{2g} = 0.051 \frac{\text{s}^2}{\text{m}}$$

7. $v_{\text{max}} \propto \sqrt{r};\ v_{\text{max}} = k\sqrt{r}$

Maximum speed is directly proportional to the square root of the track radius.

$$k_{\text{experimental}} = \frac{\Delta v_{\text{max}}}{\Delta \sqrt{r}} = 2.70 \frac{\text{m}^{\frac{1}{2}}}{\text{s}}$$

Using Newton's second law and centripetal acceleration:

$$k = \sqrt{\mu g};\ \mu = k^2/g = 0.74$$

8. $F_{\text{g}} \propto \frac{1}{r^2};\ F_{\text{g}} = \frac{k}{r^2}$

Gravitational force is inversely proportional to the square of distance.

$$k_{\text{experimental}} = \frac{\Delta F_{\text{g}}}{\Delta \frac{1}{r^2}} = 3.19 \times 10^{16}\ \text{N kg}^2$$

Using Newton's law of universal gravitation:

$$k_{\text{theoretical}} = GMm = 3.19 \times 10^{16}\ \text{N kg}^2$$

9. $B \propto \frac{1}{d^2};\ B = \frac{k}{d^2}$

Brightness is inversely proportional to the square of distance.

$$k_{\text{experimental}} = \frac{\Delta B}{\Delta \frac{1}{d^2}} = 4.58\ \text{W}$$

Using the brightness equation:

$$k_{\text{theoretical}} = \frac{L}{4\pi} = 4.77\ \text{W}$$

10. $T \propto r^{\frac{3}{2}};\ T = kr^{\frac{3}{2}}$

Orbital period is directly proportional to orbital radius to the power of 3/2.

$$k_{\text{experimental}} = \frac{\Delta T}{\Delta r^{\frac{3}{2}}} = 2.55 \times 10^{-7} \frac{\text{s}}{\text{m}^{\frac{3}{2}}}$$

Using Newton's law of universal gravitation and centripetal acceleration:

$$k = \frac{2\pi}{\sqrt{GM}};\ M = \frac{4\pi^2}{Gk^2} = 9.10 \times 10^{24}\ \text{kg}$$