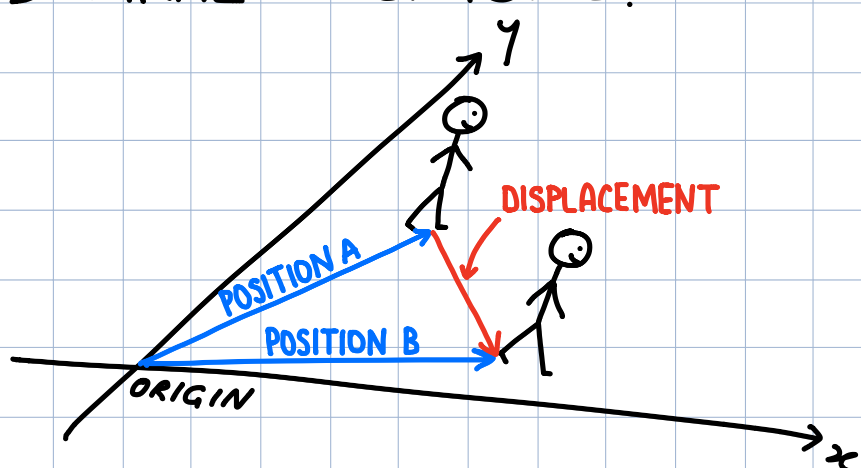
# KINEMATICS

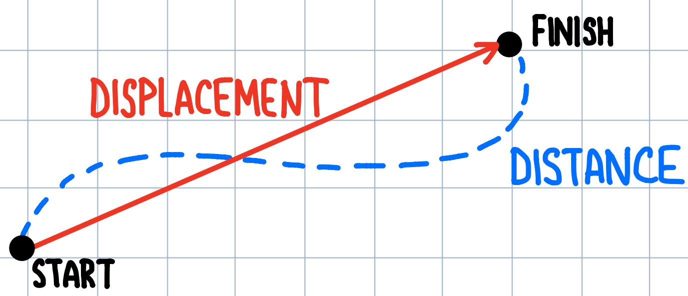
- KINEMATICS IS THE STUDY OF MOTION.

## POSITION, DISPLACEMENT AND DISTANCE

- POSITION IS A VECTOR DESCRIBING WHERE AN OBJECT IS. RELATIVE TO AN ORIGIN
- DISPLACEMENT IS THE CHANGE IN POSITION AND IS REPRESENTED BY A VECTOR JOINING THE INITIAL AND FINAL POSITIONS.

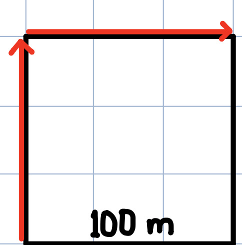



- DISTANCE IS THE LENGTH OF A PATH.
- DISTANCE IS A SCALAR.
- IF A PATH TAKEN IS STRAIGHT, THE DISTANCE IS EQUAL TO THE DISPLACEMENT.




## EXAMPLE

A DOG WALKS FROM CORNER TO CORNER AROUND A SQUARE CITY BLOCK OF SIDE LENGTH 100 m. CALCULATE THE DISTANCE TRAVELLED AND THE DISPLACEMENT.




## SPEED AND VELOCITY

- SPEED IS THE RATE AT WHICH DISTANCE IS TRAVELLED.
- SPEED IS A SCALAR.

$$\boxed{v = \frac{d}{t}}$$

IF SPEED IS NOT CONSTANT, THIS EQUATION GIVES AVERAGE SPEED

v : SPEED ($\frac{m}{s}$)
d : DISTANCE (m)
t : TIME (s)

- VELOCITY IS THE RATE AT WHICH AN OBJECT CHANGES ITS POSITION.
- VELOCITY IS A VECTOR.

$$\boxed{\vec{v} = \frac{\vec{d}}{t}}$$

$\vec{v}$ : VELOCITY ($\frac{m}{s}$)
$\vec{d}$ : DISPLACEMENT (m)
$t$ : time (s)

IF VELOCITY IS NOT CONSTANT, THIS EQUATION GIVES AVERAGE VELOCITY

- UNIFORM MOTION REFERS TO MOTION AT A CONSTANT VELOCITY.

EXAMPLE

LIGHT FROM THE SUN TAKES $8\frac{1}{2}$ MINUTES TO REACH EARTH. IF LIGHT TRAVELS AT $3.00 \times 10^8 \frac{m}{s}$, HOW FAR AWAY IS THE SUN?

## EXAMPLE

A STUDENT COMPLETES FOUR LAPS AROUND A 400 m TRACK IN 390 s. WHAT ARE THE STUDENT'S AVERAGE SPEED AND VELOCITY?

## ACCELERATION

- ACCELERATION IS THE RATE OF CHANGE OF VELOCITY.
- ACCELERATION IS A VECTOR.

$$\boxed{\vec{a} = \frac{\Delta\vec{v}}{t} = \frac{\vec{v}_f - \vec{v}_i}{t}}$$

DELTA MEANING CHANGE IN (FINAL MINUS INITIAL)

$\vec{a}$ : ACCELERATION ($\frac{m}{s^2}$)

$\Delta\vec{v}$ : CHANGE IN VELOCITY ($\frac{m}{s}$)

$\vec{v}_f$ : FINAL VELOCITY ($\frac{m}{s}$)

$\vec{v}_i$ : INITIAL VELOCITY ($\frac{m}{s}$)

$t$ : TIME (s)

- FOR MOTION WITH CONSTANT ACCELERATION, THE AVERAGE VELOCITY CAN BE EXPRESSED IN THE FOLLOWING FORMS:

$$\boxed{\bar{v} = \frac{\vec{d}}{t}}$$

$$\boxed{\bar{v} = \frac{\vec{v}_i + \vec{v}_f}{2}}$$

IF ACCELERATION IS NOT CONSTANT, THIS FORMULA DOES NOT APPLY

$\bar{v}$ : AVERAGE VELOCITY $\left(\frac{m}{s}\right)$
$\vec{v}_i$ : INITIAL VELOCITY $\left(\frac{m}{s}\right)$
$\vec{v}_f$ : FINAL VELOCITY $\left(\frac{m}{s}\right)$
$\vec{d}$ : DISPLACEMENT (m)
$t$ : TIME (s)

- USING THE THREE BASIC EQUATIONS FOR MOTION WITH CONSTANT ACCELERATION, WE CAN FORM FOUR MORE USEFUL EQUATIONS.

$$\boxed{\begin{aligned} \vec{v}_f &= \vec{v}_i + \vec{a}t \\ \vec{d} &= \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)t \\ v_f^2 &= v_i^2 + 2\vec{a}\vec{d} \\ \vec{d} &= v_i t + \tfrac{1}{2}\vec{a}t^2 \end{aligned}}$$

$\vec{a}$ : ACCELERATION $\left(\frac{m}{s^2}\right)$
$\vec{v}_f$ : FINAL VELOCITY $\left(\frac{m}{s}\right)$
$\vec{v}_i$ : INITIAL VELOCITY $\left(\frac{m}{s}\right)$
$\vec{d}$ : DISPLACEMENT (m)
$t$ : TIME (s)

# EXAMPLE

A SPRINTER, STARTING FROM REST, ACCELERATES AT A RATE OF 1.1 $\frac{m}{s^2}$ FOR THE ENTIRETY OF A 100 m DASH. WHAT IS HIS FINAL VELOCITY?

1. IDENTIFY YOUR UNKNOWN ALONG WITH <u>THREE</u> GIVENS.

2. CHOOSE THE EQUATION WITH THE UNKNOWN AND THREE GIVENS.

3. SIMPLIFY BY PLUGGING IN ANY ZEROS.

4. ALGEBRAICALLY SOLVE FOR THE UNKNOWN.

5. PLUG IN VALUES.

## EXAMPLE

RAMMUS IS POWERBALLING WITH AN ACCELERATION OF 1.4 $\frac{m}{s^2}$. IF HE STARTS FROM REST AND POWERBALLS FOR 7.0 s, HOW FAR DOES HE TRAVEL?

## EXAMPLE

A POLICE CAR INCREASES ITS SPEED FROM 50 $\frac{km}{h}$ TO 90 $\frac{km}{h}$ IN 2.5 s. WHAT WAS ITS ACCELERATION?

# FREE FALL

- ACCELERATION DUE TO GRAVITY, $g$, IS THE ACCELERATION OF FREE FALL DUE TO GRAVITATIONAL FORCE.
- ON THE SURFACE OF EARTH, $g = 9.8 \frac{m}{s^2}$ DOWN. CAN BE POSITIVE OR NEGATIVE (SEE ① BELOW)

## PROBLEM SOLVING TIPS

① BEFORE LISTING YOUR GIVENS, DECIDE ON ONE DIRECTION TO BE POSITIVE. ALL VECTORS IN THE OPPOSITE DIRECTION WILL BE NEGATIVE.


② CAN'T FIND YOUR THREE GIVENS? HERE ARE SOME NOT EXPLICITLY STATED:

- FREE FALL: $a = 9.8 \frac{m}{s^2}$ DOWN
- STARTS AT REST: $v_i = 0 \frac{m}{s}$
- OBJECT IS DROPPED FROM A STATIONARY POSITION: $v_i = 0 \frac{m}{s}$
- OBJECT IN FREE FALL REACHES ITS PEAK: $v_f = 0 \frac{m}{s}$

- OBJECT IN FREE FALL INITIALLY MOVING UPWARDS RETURNS TO ORIGINAL POSITION/HEIGHT : $v_f = -v_i$

③ IF YOU ARE ASKED FOR THE VELOCITY AT WHICH AN OBJECT HITS THE GROUND, YOU ARE LOOKING FOR THE VELOCITY BEFORE STOPPING : $v_f \neq 0 \frac{m}{s}$

## EXAMPLE

HOW FAR WILL A HAMMER FALL AFTER 2.0 s IF IT IS DROPPED FROM REST ?

## EXAMPLE

A BULLET IS FIRED FROM A GUN UPWARDS AT 700 $\frac{m}{s}$. WHAT MAXIMUM HEIGHT WILL IT REACH?

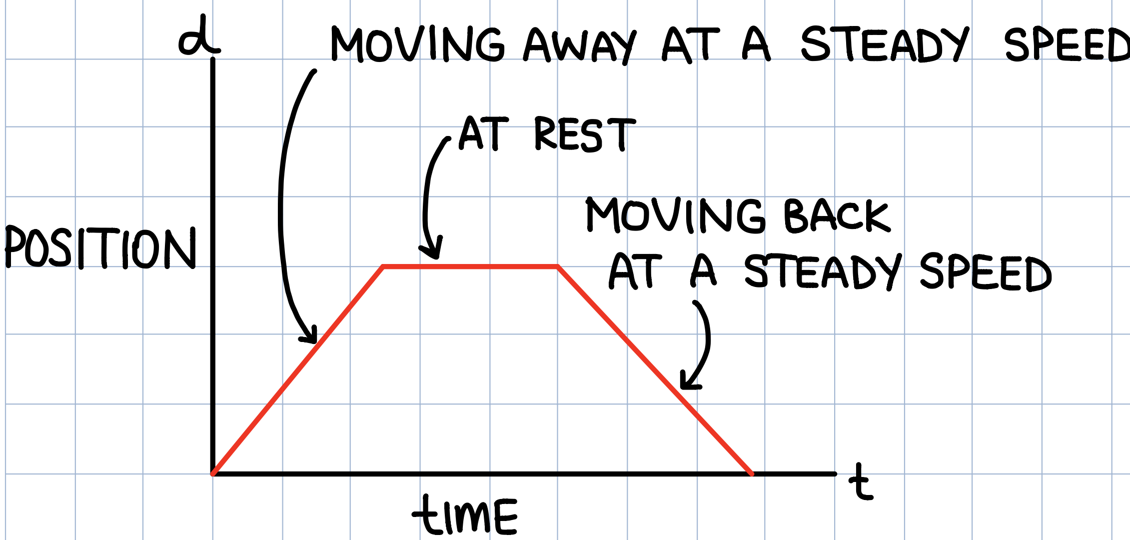
## EXAMPLE

A BOY THROWS A BALL UPWARDS AT A SPEED OF 15 $\frac{m}{s}$. HOW LONG DOES IT TAKE TO RETURN TO HIS HAND?

# GRAPHS OF MOTION

## POSITION vs. TIME GRAPHS

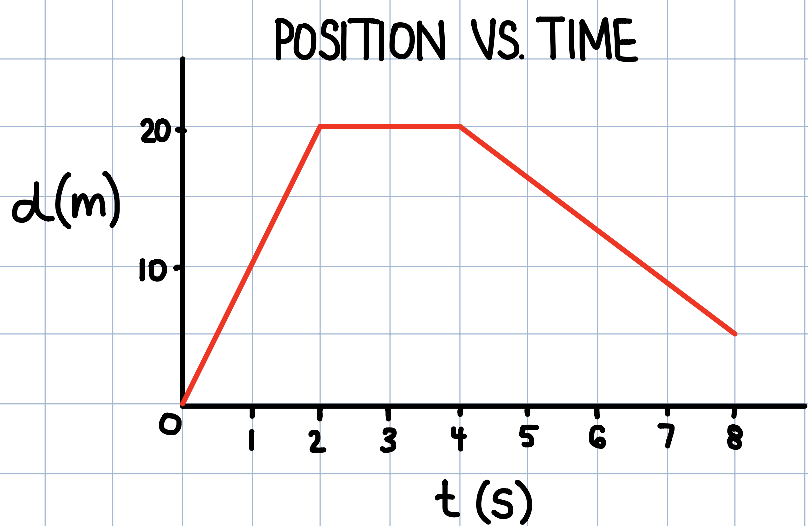
OR DISPLACEMENT VS. TIME



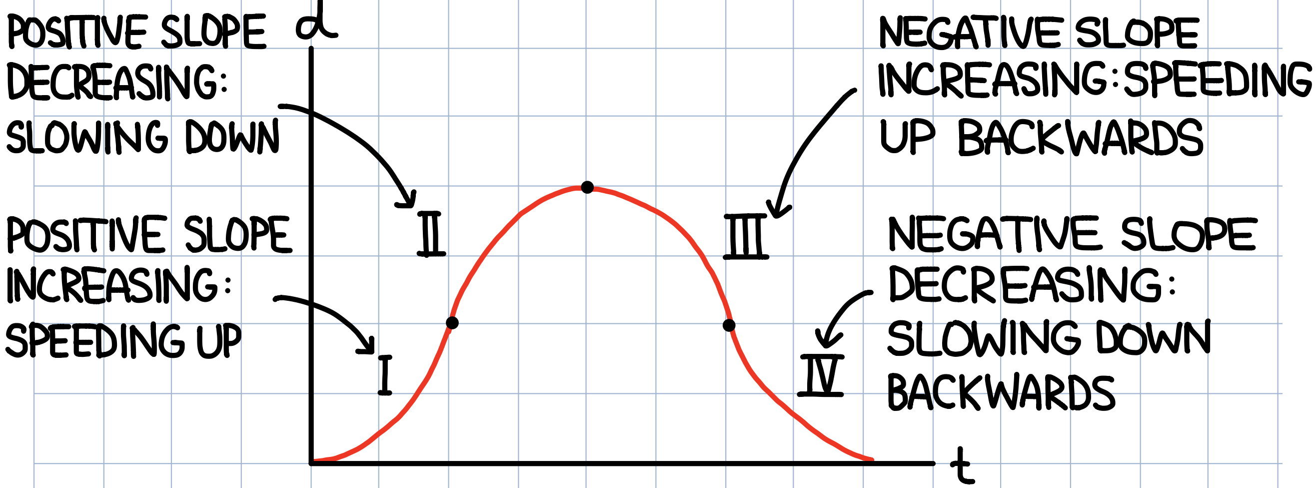

- THE SLOPE OF A d vs. t GRAPH IS EQUAL TO THE VELOCITY.
- THE AVERAGE VELOCITY IS THE SLOPE OF A STRAIGHT LINE JOINING TWO POINTS ON A d vs. t GRAPH.

EXAMPLE

CALCULATE THE VELOCITY FOR EACH PART OF THE GRAPH.

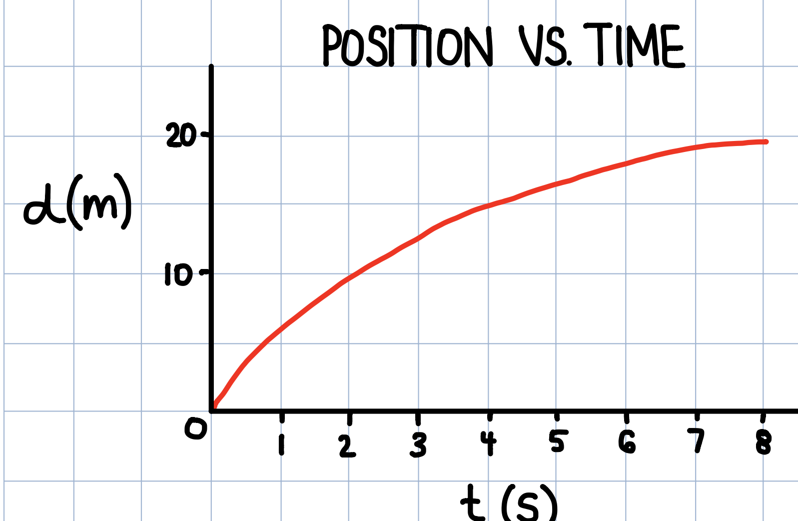



- A CURVE ON A $d$ vs. $t$ GRAPH DESCRIBES ACCELERATION.
- FOR A CURVED $d$ vs. $t$ GRAPH, ESTIMATE THE SLOPE USING A TANGENT LINE.
- THE SLOPE OF A TANGENT LINE ON A $d$ vs. $t$ GRAPH IS THE INSTANTANEOUS VELOCITY.

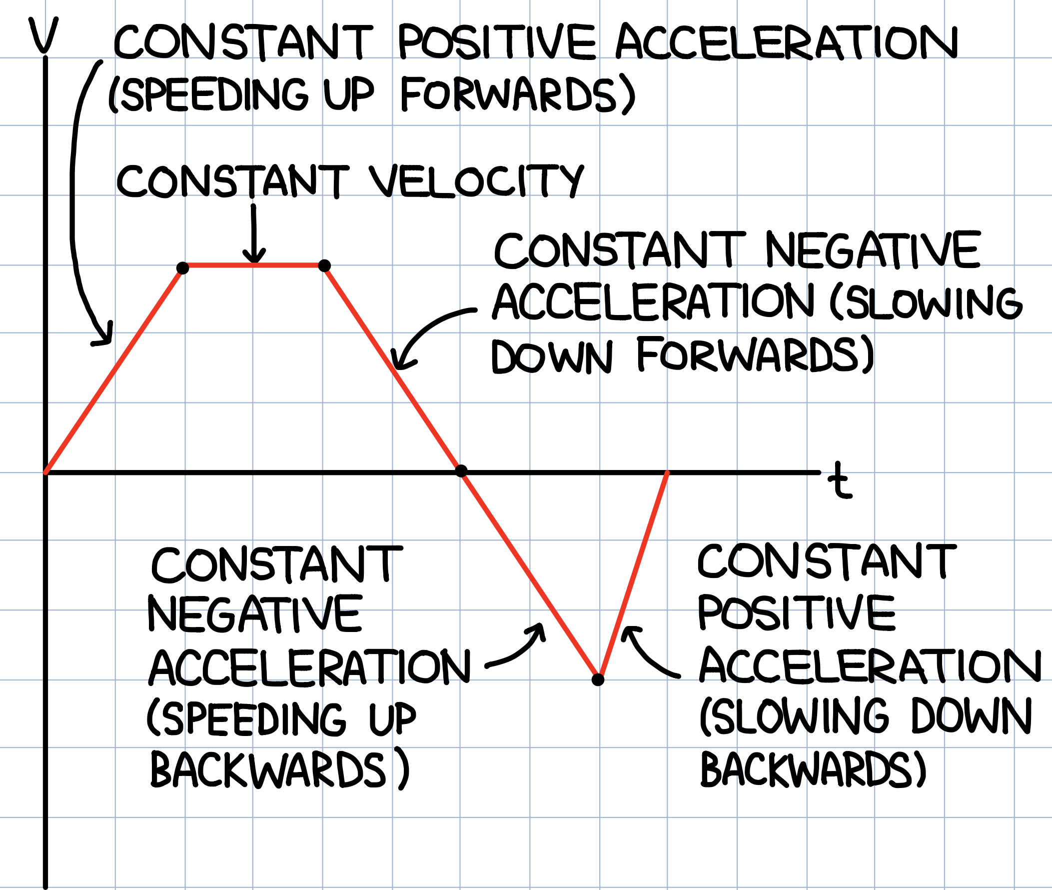



## EXAMPLE

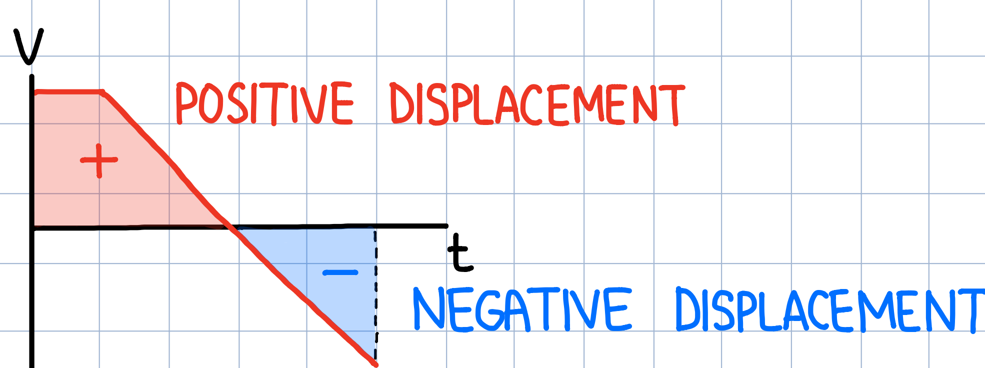
DETERMINE THE INSTANTANTANEOUS VELOCITY AT 4.0 s AND THE AVERAGE VELOCITY FROM 0 TO 8.0 s.



# VELOCITY vs. TIME GRAPHS



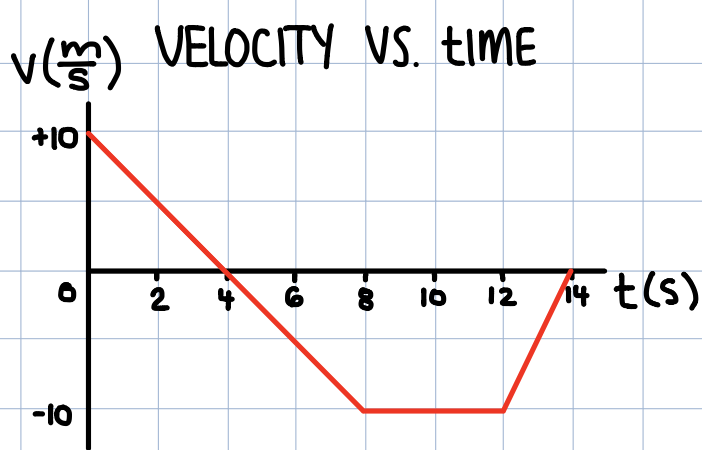

- THE SLOPE OF A v vs. t GRAPH IS EQUAL TO THE ACCELERATION.
- THE AREA UNDER A v vs. t GRAPH IS EQUAL TO THE DISPLACEMENT.



## EXAMPLE

USE THE GRAPH TO CALCULATE THE FOLLOWING:

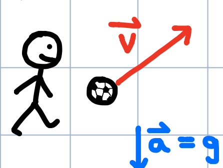
a) THE ACCELERATION AT $t=4s$
b) THE AVERAGE ACCELERATION FROM 0 TO 14 s
c) THE TOTAL DISPLACEMENT
d) THE TOTAL DISTANCE TRAVELLED
e) THE AVERAGE VELOCITY FROM 0 TO 14 s
f) THE AVERAGE SPEED FROM 0 TO 14 s

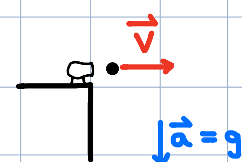


# PROJECTILE MOTION

- **PROJECTILE MOTION** IS ANY SORT OF FREE FALL MOTION THAT HAS A HORIZONTAL COMPONENT OF VELOCITY.

*EXAMPLE*




- THE MOTION OF A PROJECTILE CAN BE ANALYZED BY LOOKING AT THE HORIZONTAL AND VERTICAL COMPONENTS SEPARATELY. EACH DIRECTION IS INDEPENDENT OF THE OTHER.
  - THE HORIZONTAL COMPONENT FOLLOWS UNIFORM MOTION ($v_x$ = CONSTANT).

$$d_x = v_x t$$

- THE VERTICAL COMPONENT FOLLOWS UNIFORMLY ACCELERATED MOTION ($a_y = 9.8\ \frac{m}{s^2}$ DOWN).

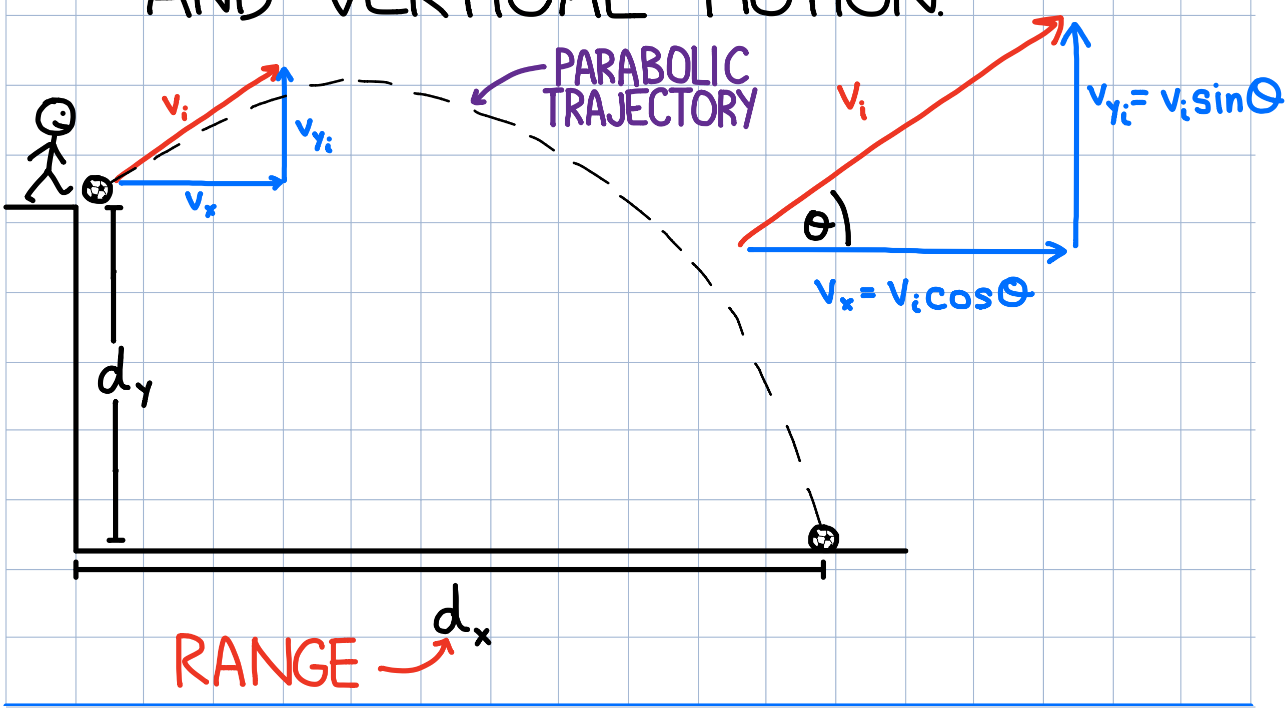
$$V_{y_f} = V_{y_i} + a_y t$$

$$d_y = \left(\frac{V_{y_i} + V_{y_f}}{2}\right) t$$

$$V_{y_f}^2 = V_{y_i}^2 + 2 a_y d_y$$

$$d_y = v_{y_i} t + \frac{1}{2} a_y t^2$$

- TIME LINKS THE HORIZONTAL AND VERTICAL MOTION.



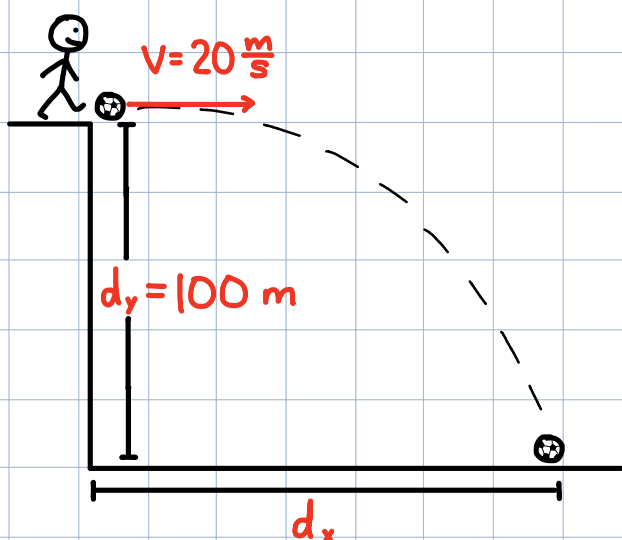
## EXAMPLE

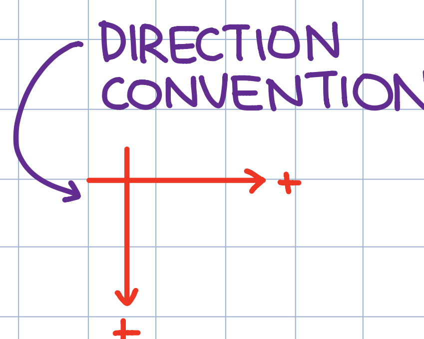
A SOCCER BALL IS KICKED OFF OF A 100 m CLIFF WITH A VELOCITY OF 20 $\frac{m}{s}$ TO THE RIGHT.

a) HOW LONG WILL IT SPEND IN THE AIR?

b) HOW FAR FROM THE BASE OF THE CLIFF WILL IT LAND?

c) WHAT WILL BE ITS VELOCITY RIGHT BEFORE IT HITS THE GROUND?




| | HORIZONTAL | VERTICAL |
|---|---|---|
| a) | $V_x = 20\frac{m}{s}$ | $V_{y_i} = 0$ |
| | $t = ?$ | $a_y = +9.8\frac{m}{s^2}$ |
| | $d_x = ?$ | $d_y = +100$ m |
| | | $t = ?$ |

$v_{y_i} = 0$

$d = \cancel{v_i}t + \frac{1}{2}at^2$

$d = \frac{1}{2}at^2$

$t^2 = \frac{2d}{a}$

$t = \sqrt{\frac{2d}{a}}$

$= \sqrt{\frac{2(100)}{9.8}}$

$= 4.5175\ s \rightarrow \boxed{4.5\ s}$

b) $v_x = 20\ \frac{m}{s}$

$t = 4.5175\ s$

$d_x = ?$

$d = vt$

$= (20)(4.5175)$

$= 90.3508\ m$

$\rightarrow 90.\ m$

c) $v_x = 20\ \frac{m}{s}$

$t = 4.5175\ s$

$d_x = 90.3508\ m$

$v_{y_i} = 0$

$a_y = +9.8\ \frac{m}{s^2}$

$d_y = +100\ m$

$t = 4.5175\ s$

$v_{yf} = ?$

$$V_{y_i} = 0$$

$$V_f^2 = \cancel{V_i^2} + 2ad$$

$$V_f^2 = 2ad$$

$$V_f = \sqrt{2ad}$$

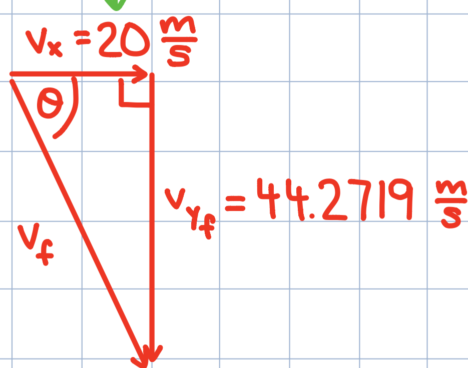
$$= \sqrt{2(9.8)(100)}$$

$$= +44.2719 \ \tfrac{m}{s}$$

(DOWN)

$V_x = 20 \ \frac{m}{s}$ RIGHT

$V_{y_f} = 44.2719 \ \frac{m}{s}$ DOWN




$$V_f^2 = V_x^2 + V_{y_f}^2$$

$$V_f = \sqrt{V_x^2 + V_{y_f}^2}$$

$$= \sqrt{(20)^2 + (44.2719)^2}$$

$$= 48.5798 \ \tfrac{m}{s}$$

$$\tan\theta = \frac{V_{y_f}}{V_x}$$

$$\theta = \tan^{-1}\left(\frac{V_{y_f}}{V_x}\right)$$

$$= \tan^{-1}\left(\frac{44.2719}{20}\right)$$

$$= 65.69^\circ$$

$49 \ \frac{m}{s}$ $66^\circ$ BELOW THE HORIZONTAL

EXAMPLE

A BALL IS THROWN FROM LEVEL GROUND WITH A VELOCITY OF $12\,\frac{m}{s}$ $60°$ ABOVE THE HORIZONTAL.

a) HOW LONG WILL IT SPEND IN THE AIR?

b) HOW FAR FROM THE THROWER WILL IT LAND?

c) WHAT WILL BE ITS VELOCITY RIGHT BEFORE IT HITS THE GROUND?