

## More Relationships Between Variables - Solutions

1. Consider the equation for the volume of a cylinder

$$V = \pi r^2 h$$

where  $r$  represents the radius and  $h$  represents the height.

- a) Determine the relationship between volume,  $V$ , and radius,  $r$ . Express the relationship in both words and symbols. **Volume is directly proportional to the square of radius.  $V \propto r^2$**   
b) Determine the relationship between volume,  $V$ , and height,  $h$ . Express the relationship in both words and symbols. **Volume is directly proportional to height.  $V \propto h$**

Determine the change in volume for each of the following changes.

- c) The height is increased by a factor of four. **4x**  
d) The radius is halved. **1/4x**  
e) The radius is decreased by a factor of three and the height is doubled. **2/9x**

A cylindrical glass can hold 400 mL of water. Determine how much water the glass can hold for each of the following changes.

- f) The height is tripled. **1200 mL**  
g) The radius is doubled. **1600 mL**  
h) The radius is halved and the height is decreased by a factor of four. **25 mL**

2. Consider the equation for magnetic field around a current-carrying wire

$$B = \frac{\mu_0 I}{2\pi d}$$

where  $\mu_0$  represents the permeability of free space (a constant),  $I$  represents the the current through the wire and  $d$  represents distance from the wire.

- a) Determine the relationship between magnetic field,  $B$ , and current,  $I$ . Express the relationship in both words and symbols.  
**Magnetic field is directly proportional to current.  $B \propto I$**   
b) Determine the relationship between magnetic field,  $B$ , and distance,  $d$ . Express the relationship in both words and symbols.  
**Magnetic field is inversely proportional to distance.  $B \propto \frac{1}{d}$**

Determine the change in magnetic field for each of the following changes.

- c) The current is halved. **1/2x**  
d) The distance from the wire is decreased by a factor of five. **5x**  
e) The current is increased by a factor of ten and the distance from the wire is tripled. **10/3x**

A long wire carries a current of 100 mA. At a distance  $x$  from the wire, the magnetic field is found to be 20 nT. Determine the magnetic field for each of the following changes.

- f) The current is decreased to 25 mA. **5nT**  
g) The distance from the wire is increased to  $5x$ . **4 nT**  
h) The current is increased to 300 mA and the distance from the wire is decreased to  $x/4$ .  
**240 nT**

3. Consider the equation for the period of a mass-spring oscillator

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where  $m$  represents the mass and  $k$  represents the spring constant.

- a) Determine the relationship between period,  $T$ , and mass,  $m$ . Express the relationship in both words and symbols.

Period is directly proportional to the square root of mass.  $T \propto \sqrt{m}$

- b) Determine the relationship between period,  $T$ , and the spring constant,  $k$ . Express the relationship in both words and symbols.

Period is inversely proportional to the square root of the spring constant.  $T \propto \frac{1}{\sqrt{k}}$

Determine the change in period for each of the following changes.

- c) The mass is decreased by a factor of four.  $1/2\times$   
d) The spring constant is increased by a factor of nine.  $1/3\times$   
e) The mass and spring constant are both tripled. **no change**

A mass attached to a spring oscillates with a period of 0.80 seconds. Determine the period for each of the following changes.

- f) The spring constant is decreased by a factor of four. **1.6 s**  
g) The mass is increased by a factor of 25. **4.0 s**  
h) The mass is halved and the spring constant is increased by a factor of eight. **0.20 s**

4. Consider the equation for the electric force between two charges

$$F_e = k \frac{q_1 q_2}{r^2}$$

where  $k$  represents the electrostatic constant,  $q_1$  and  $q_2$  represent the charges and  $r$  represents the separation distance.

Two charges are separated by a distance of 20 mm. The electric force at this distance is 2 N. Determine the electric force between the charges for the following changes.

- a) One charge is halved. **1 N**  
b) Both charges are increased by a factor of three. **18 N**  
c) The distance separating the charges is increased to 100 mm. **0.08 N**  
d) The distance separating the charges is decreased to 10 mm. **8 N**  
e) The distance separating the charges is decreased to 50 mm. **0.32 N**  
f) One charge is halved and the distance separating the charges is decreased to 10 mm. **4 N**  
g) Both charges are increased by a factor of ten and the distance separating the masses is increased to 100 mm. **8 N**  
h) One charge is doubled, the other is decreased by a factor of five, and the distance separating them is decreased to 4 mm. **20 N**