

ORBITS - SOLUTIONS

1. GIVEN:

$$m = 1500 \text{ kg}$$

$$M = 5.98 \times 10^{24} \text{ kg}$$

$$R = R_E + 4.0 \times 10^5 \text{ m}$$

$$= 6.78 \times 10^6 \text{ m}$$

$$v = ?$$

$$F_c = ma_c$$

$$F_g = ma_c$$

$$G \frac{Mm}{R^2} = m \frac{v^2}{R}$$

$$v = \sqrt{G \frac{M}{R}}$$

$$= \sqrt{(6.67 \times 10^{-11}) \frac{5.98 \times 10^{24}}{6.78 \times 10^6}}$$

$$= 7670 \frac{\text{m}}{\text{s}}$$

2. a) GIVEN:

$$T = 1.4 \times 10^6 \text{ s}$$

$$R = 1.2 \times 10^9 \text{ m}$$

$$a_c = ?$$

$$a_c = \frac{4\pi^2 R}{T^2}$$

$$= \frac{4\pi^2 (1.2 \times 10^9)}{(1.4 \times 10^6)^2}$$

$$= 0.024 \frac{\text{m}}{\text{s}^2}$$

b) GIVEN:

$$T = 1.4 \times 10^6 \text{ s}$$

$$R = 1.2 \times 10^9 \text{ m}$$

$$M = ?$$

$$F_c = ma_c$$

$$F_g = ma_c$$

$$G \frac{Mm}{R^2} = m \frac{4\pi^2 R}{T^2}$$

$$M = \frac{4\pi^2 R^3}{GT^2}$$

$$= \frac{4\pi^2 (1.2 \times 10^9)^3}{(6.67 \times 10^{-11}) (1.4 \times 10^6)^2}$$

$$= 5.2 \times 10^{26} \text{ kg}$$

3. GIVEN:

$$v = 3.1 \times 10^3 \frac{\text{m}}{\text{s}}$$

$$M = 5.98 \times 10^{24} \text{ kg}$$

$$R = ?$$

$$F_c = ma_c$$

$$F_g = ma_c$$

$$G \frac{Mm}{R^2} = m \frac{v^2}{R}$$

$$R = G \frac{M}{v^2}$$

$$= (6.67 \times 10^{-11}) \frac{(5.98 \times 10^{24})}{(3.1 \times 10^3)^2}$$

$$= 4.2 \times 10^7 \text{ m}$$

4. a) GIVEN:

$$M = 1.31 \times 10^{22} \text{ kg}$$

$$R = 17500 \text{ km}$$

$$= 1.75 \times 10^7 \text{ m}$$

$$v = ?$$

$$F_c = ma_c$$

$$F_g = ma_c$$

$$G \frac{Mm}{R^2} = m \frac{v^2}{R}$$

$$v = \sqrt{G \frac{M}{R}}$$

$$= \sqrt{(6.67 \times 10^{-11}) \frac{1.31 \times 10^{22}}{1.75 \times 10^7}}$$

$$= 223 \frac{\text{m}}{\text{s}}$$

b) GIVEN:

$$M = 1.31 \times 10^{22} \text{ kg}$$

$$R = 17500 \text{ km}$$

$$= 1.75 \times 10^7 \text{ m}$$

$$T = ?$$

$$F_c = ma_c$$

$$F_g = ma_c$$

$$G \frac{Mm}{R^2} = m \frac{4\pi^2 R}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}}$$

$$= \sqrt{\frac{4\pi^2 (1.75 \times 10^7)^3}{(6.67 \times 10^{-11})(1.31 \times 10^{22})}}$$

$$= 4.92 \times 10^5 \text{ s}$$

5. GIVEN:

$$M = 4.44 \times 10^{23} \text{ kg}$$

$$T = 6.0 \times 10^5 \text{ s}$$

$$R = ?$$

$$F_c = ma_c$$

$$F_g = ma_c$$

$$G \frac{Mm}{R^2} = m \frac{4\pi^2 R}{T^2}$$

$$R = \sqrt[3]{G \frac{MT^2}{4\pi^2}}$$

$$= \sqrt[3]{\frac{(6.67 \times 10^{-11}) (4.44 \times 10^{23}) (6.0 \times 10^5)^2}{4\pi^2}}$$

$$= 6.5 \times 10^7 \text{ m}$$

6.

$$\begin{aligned}F_c &= ma_c \\F_g &= ma_c \\G \frac{Mm}{R^2} &= m \frac{4\pi^2 R}{T^2} \\T &= \sqrt{\frac{4\pi^2 R^3}{GM}}\end{aligned}$$

$$\begin{aligned}F_c &= ma_c \\F_g &= ma_c \\G \frac{Mm}{R^2} &= m \frac{v^2}{R} \\v &= \sqrt{G \frac{M}{R}}\end{aligned}$$

a) T HAS NO m
DEPENDENCE.

V HAS NO m
DEPENDENCE.

T

V

b) $T \propto \sqrt{R^3}$

$$v \propto \sqrt{\frac{1}{R}}$$

$$R' = 3R$$

$$R' = 3R$$

$$\begin{aligned}T' &= \sqrt{3^3} T \\&= 3\sqrt{3} T \\&= 5.196 T\end{aligned}$$

$$\begin{aligned}v' &= \sqrt{\frac{1}{3}} v \\&= \frac{1}{\sqrt{3}} v \\&= 0.5774 v\end{aligned}$$

$$c) T \propto \sqrt{R^3}$$

$$R' = \frac{1}{2}R$$

$$\begin{aligned} T' &= \sqrt{\left(\frac{1}{2}\right)^3} T \\ &= \frac{1}{2\sqrt{2}} T \\ &= 0.3536 T \end{aligned}$$

$$v \propto \sqrt{\frac{1}{R}}$$

$$R' = \frac{1}{2}R$$

$$\begin{aligned} v &= \sqrt{\frac{1}{\frac{1}{2}}} v \\ &= \sqrt{2} v \\ &= 1.4142 v \end{aligned}$$