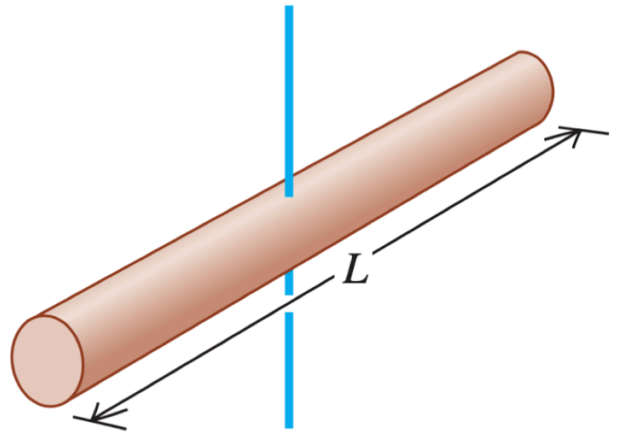


## Rotational Inertia Calculations

Uniform rod of mass  $M$  and length  $L$  about its center

Determine the **linear mass density**  $\lambda$  of the rod.

Divide the rod into small segments of length  $dx$ . Take one of these infinitesimally small segments of the rod located a distance  $x$  from the center, where  $x = 0$ .

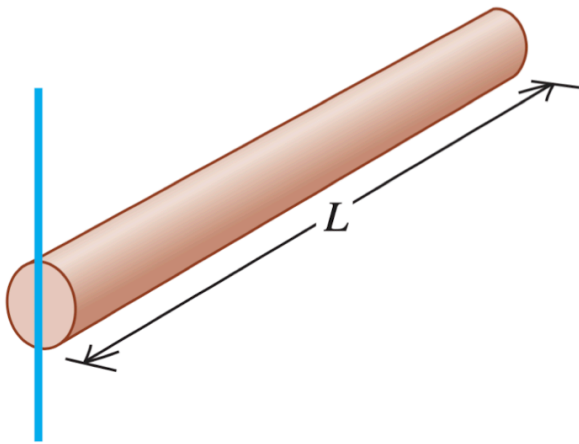


Determine the mass  $dm$  of this segment.

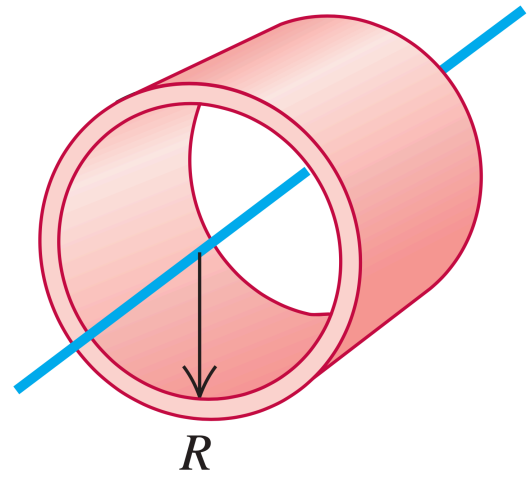
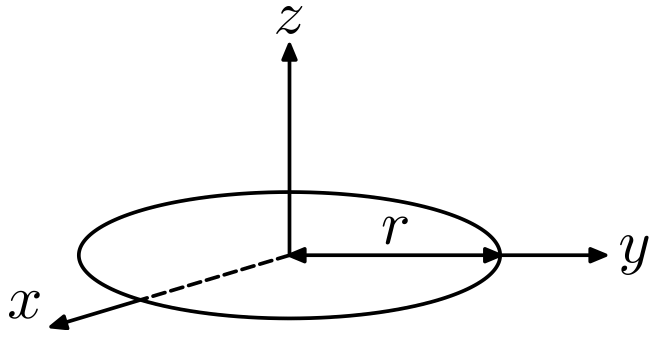
Determine the rotational inertia  $dI$  of this segment for an axis perpendicular to the rod and through its center of mass.

Integrate to get the rotational inertia  $I$  of the rod due to all the segments.

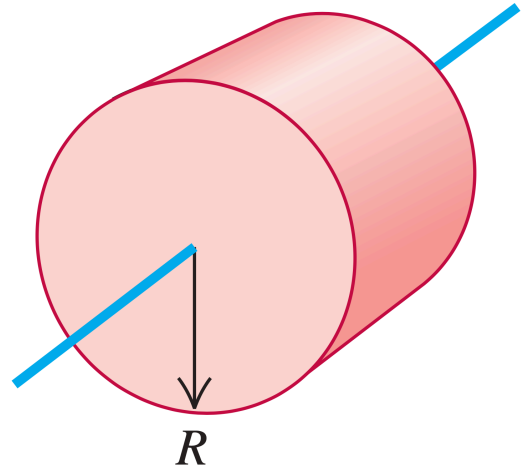
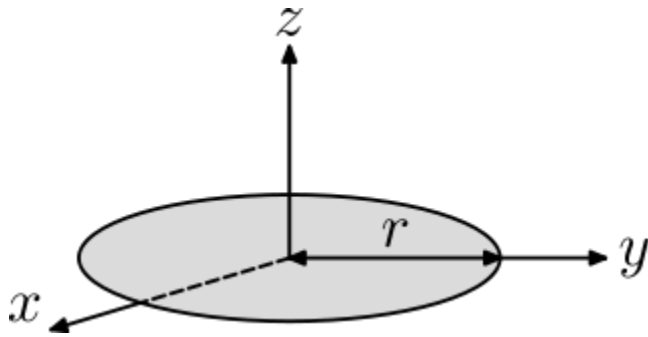
Uniform rod of mass  $M$  and length  $L$  about one end



Thin loop of mass  $M$  and radius  $R$  (or thin cylindrical shell)



Uniform solid disk of mass  $M$  and radius  $R$  (or uniform solid cylinder)



Determine the **area mass density**  $\sigma$  of the disk.

Divide the disk into thin loops of width  $dr$ . Take one of these infinitesimally thin loops with radius  $r$ .

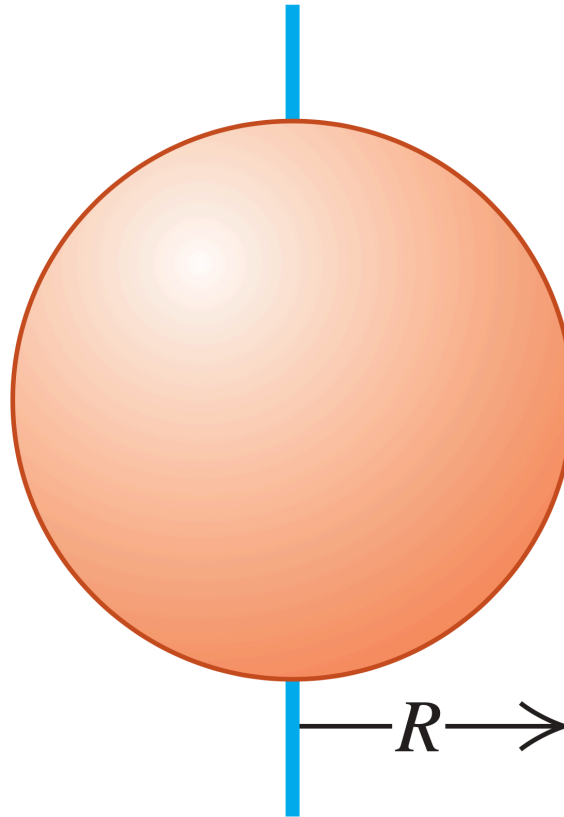
Determine the area  $dA$  of this loop.

Determine the mass  $dm$  of this loop.

Determine the rotational inertia  $dI$  of this loop.

Integrate to get the rotational inertia  $I$  of the disk due to all the loops.

Uniform spherical shell of mass  $M$  and radius  $R$



Determine the **area mass density**  $\sigma$  of the spherical shell.

Divide the shell into thin loops of width  $ds$ . Take one of these infinitesimally thin loops with radius  $r$ .

Determine the radius of this loop  $r$  in terms of  $\phi$ .

Determine the width  $ds$  of this loop in terms of  $d\phi$ .

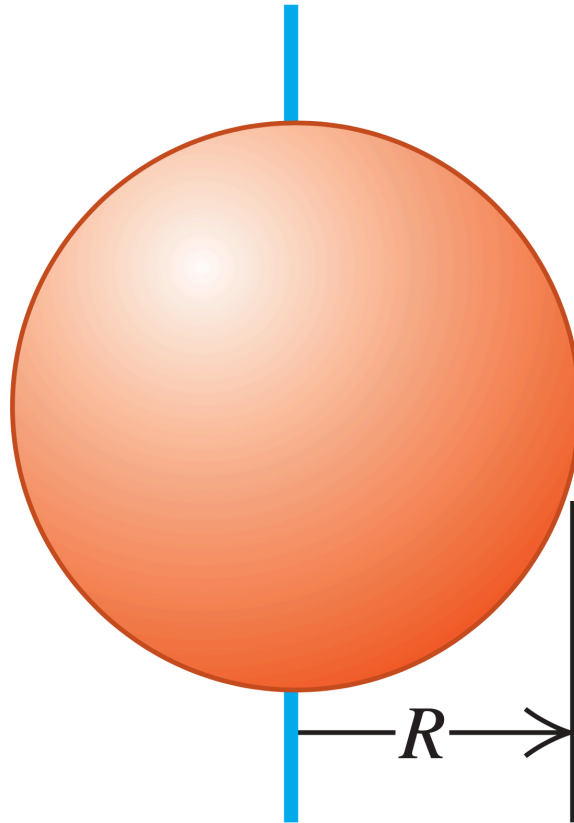
Determine the area  $dA$  of this loop.

Determine the mass  $dm$  of this loop.

Determine the rotational inertia  $dI$  of this loop.

Integrate to get the rotational inertia  $I$  of the spherical shell due to all the loops.

Uniform Solid Sphere of mass  $M$  and radius  $R$



Determine the **volume mass density**  $\rho$  of the sphere.

Divide the sphere into thin spherical shells of thickness  $dr$ .  
Take one of these infinitesimally thin spherical shells with  
radius  $r$ .

Determine the volume  $dV$  of this spherical shell.

Determine the mass  $dm$  of this spherical shell.



Determine the rotational inertia  $dI$  of this spherical shell.

Integrate to get the rotational inertia  $I$  of the sphere due to all the spherical shells.