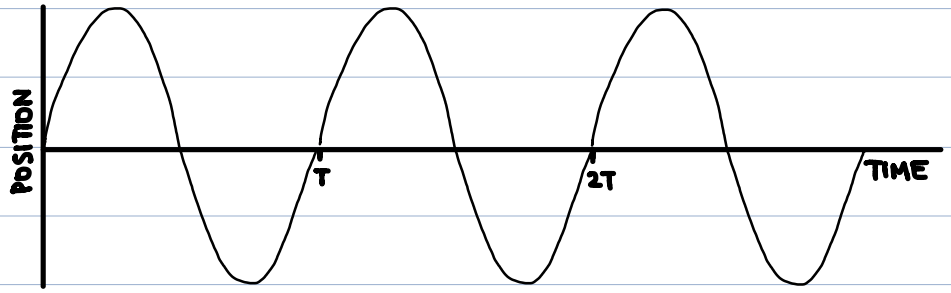
SIMPLE HARMONIC MOTION

- SIMPLE HARMONIC MOTION IS A PERIODIC (BACK-AND-FORTH) MOTION WITH A POSITION-TIME GRAPH RESEMBLING A SINE FUNCTION.



- SIMPLE HARMONIC MOTION IS CAUSED BY A RESTORING FORCE WHICH IS DIRECTLY PROPORTIONAL TO DISPLACEMENT (e.g. HOOKE'S LAW).

$$x(t) = A\cos(\omega t)$$

x: POSITION (m)
A: AMPLITUDE (m)
ω: ANGULAR FREQUENCY ($\frac{rad}{s}$)
t: TIME (s)

- THE AMPLITUDE IS THE MAXIMUM DISPLACEMENT OF THE OBJECT FROM ITS EQUILIBRIUM POSITION.
- THE PERIOD IS THE TIME FOR ONE CYCLE OF SIMPLE HARMONIC MOTION.
- THE FREQUENCY IS THE NUMBER OF CYCLES PER SECOND.

$$T = \frac{1}{f}$$

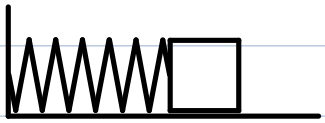
$$\omega = 2\pi f = \frac{2\pi}{T}$$

ω: ANGULAR FREQUENCY ($\frac{rad}{s}$)
f: FREQUENCY (Hz)
T: PERIOD (s)

ANGULAR FREQUENCY IS ALSO KNOWN AS ANGULAR SPEED AND IS USED IN DESCRIBING SIMPLE HARMONIC MOTION EVEN IF THE OBJECT IS NOT ROTATING.

MASS-SPRING SYSTEMS

- THE MOTION OF A MASS ATTACHED TO A SPRING IS SIMPLE HARMONIC.

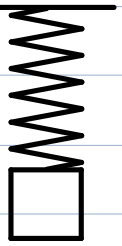


RECALL $F_s = kx$ AND $E_p = \frac{1}{2}kx^2$.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

T: PERIOD (s)
m: MASS (kg)
k: SPRING CONSTANT $(\frac{N}{m})$

A VERTICAL MASS-SPRING SYSTEM UNDER THE INFLUENCE OF GRAVITY CAN BE TREATED IN THE SAME WAY AS A HORIZONTAL MASS-SPRING SYSTEM: THE FORCE OF GRAVITY JUST CAUSES THE EQUILIBRIUM POSITION TO CHANGE.



EXAMPLE

A 0.75 kg MASS IS ATTACHED TO THE END OF A SPRING WITH A SPRING CONSTANT OF 80 $\frac{N}{m}$. THE MASS-SPRING SYSTEM IS ON A HORIZONTAL FRICTIONLESS SURFACE AND STRETCHED 30 cm TO THE RIGHT OF ITS EQUILIBRIUM POSITION.

a) HOW LONG DOES IT TAKE FROM WHEN THE MASS IS RELEASED FOR THE SYSTEM TO FIRST PASS ITS EQUILIBRIUM POSITION?

b) WHAT IS THE SPEED OF THE MASS AT THE EQUILIBRIUM POSITION?

c) WHAT IS THE MAXIMUM ACCELERATION OF THE MASS?

d) DRAW THE FOLLOWING GRAPHS OVER TWO PERIODS. USE RIGHT AS THE POSITIVE DIRECTION.

i) POSITION VS. TIME

ii) VELOCITY VS. TIME

iii) ACCELERATION VS. TIME

PENDULUMS

- THE MOTION OF A PENDULUM IS SIMPLE HARMONIC FOR SMALL AMPLITUDES (ANGLES).

$$T = 2\pi\sqrt{\frac{L}{g}}$$

T: PERIOD (s)
L: LENGTH OF PENDULUM (m)
g: ACCELERATION DUE TO GRAVITY ($9.8 \frac{m}{s^2}$)

EXAMPLE

A PENDULUM OF LENGTH 1.00m IS RAISED TO A HEIGHT OF 5 cm ABOVE ITS EQUILIBRIUM POSITION AND RELEASED FROM REST.

a) WHAT IS ITS PERIOD?
b) WHAT IS ITS SPEED AT THE LOWEST POINT?
c) IF THE PENDULUM IS INSTEAD RAISED TO A HEIGHT OF 10 cm ABOVE ITS EQUILIBRIUM POSITION, HOW WILL THE ANSWERS TO a) AND b) CHANGE?