SPECIAL RELATIVITY

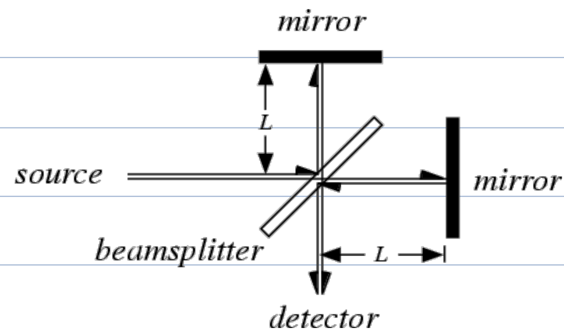
BASIC CONCEPTS

- FRAME OF REFERENCE: A COORDINATE SYSTEM IN WHICH EXPERIMENTERS MAKE POSITION AND TIME MEASUREMENTS.
- RELATIVE SPEED: THE SPEED OF ONE OBJECT COMPARED TO ANOTHER OBJECT.
  - OBJECTS ARE AT REST RELATIVE TO EACH OTHER IF THEIR VELOCITIES ARE THE SAME.
  - OBJECTS ARE MOVING RELATIVE TO EACH OTHER IF THEIR VELOCITIES ARE DIFFERENT.
- INERTIAL FRAME OF REFERENCE: ANY SYSTEM THAT MOVES AT A CONSTANT VELOCITY RELATIVE TO ANOTHER INERTIAL FRAME OF REFERENCE

- NON-INERTIAL FRAME OF REFERENCE: ANY SYSTEM THAT MOVES WITH ACCELERATION RELATIVE TO AN INERTIAL FRAME OF REFERENCE
- SPECIAL RELATIVITY: THE STUDY OF INERTIAL FRAMES OF REFERENCE
- SPEED OF LIGHT: THE FASTEST SPEED IN THE UNIVERSE, $c = 3.00 \times 10^8 \frac{m}{s}$
- RELATIVISTIC SPEED: A SPEED WHICH IS A SIGNIFICANT PROPORTION OF THE SPEED OF LIGHT SO THE CONSEQUENCES OF SPECIAL RELATIVITY MUST BE TAKEN INTO ACCOUNT (USUALLY GREATER THAN 10% OF THE SPEED OF LIGHT)

MICHELSON-MORLEY EXPERIMENT

- PURPOSE: TO FIND THE AETHER (MEDIUM) THROUGH WHICH LIGHT TRAVELS



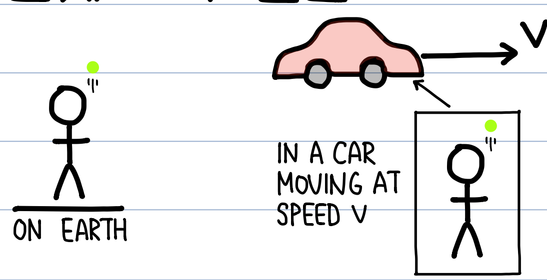
- FOUND THAT THE TWO PULSES ALWAYS MOVED WITH THE EXACT SAME SPEED
  ∴ THERE IS NO AETHER.

POSTULATES OF SPECIAL RELATIVITY

1. FIRST POSTULATE (PRINCIPLE OF RELATIVITY)

- THE LAWS OF PHYSICS ARE THE SAME IN ALL INERTIAL FRAMES OF REFERENCE.
  THERE IS NO PREFERRED FRAME OF REFERENCE.

EXAMPLE:

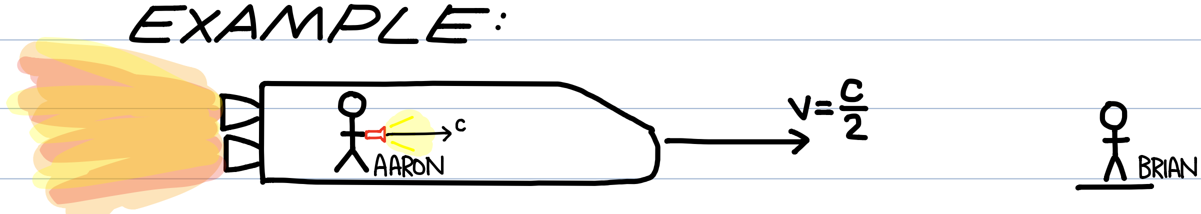


IF BOTH THROW A BALL UP WITH THE SAME INITIAL VELOCITY THE BALLS WILL BOTH LAND IN THEIR HANDS AFTER EQUAL AMOUNTS OF TIME.

2. SECOND POSTULATE

- THE SPEED OF LIGHT IS THE SAME IN ALL INERTIAL FRAMES OF REFERENCE

EXAMPLE:



AARON TURNS ON A FLASHLIGHT WHILE IN A SPACECRAFT MOVING AT SPEED $\frac{1}{2}c$ WITH RESPECT TO BRIAN. AARON AND BRIAN BOTH OBSERVE THE LIGHT TO BE TRAVELLING AT SPEED c.
BRIAN DOES NOT OBSERVE THE LIGHT TO BE TRAVELLING AT $\frac{3}{2}c$!

SIMULTANEITY

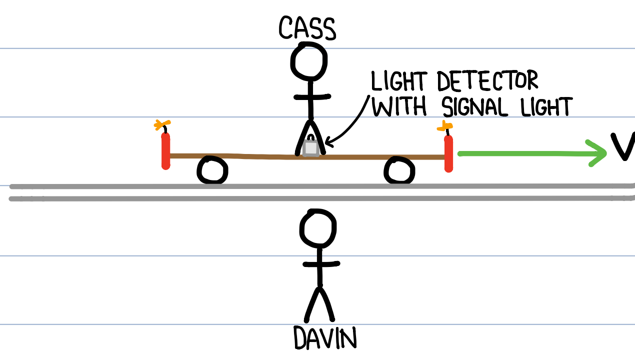
- AN EVENT IS A PHYSICAL ACTIVITY THAT OCCURS AT A DEFINITE POINT IN SPACE AND A DEFINITE INSTANT IN TIME.

  EXAMPLE

  - A FIRECRACKER EXPLODES
  - TWO PARTICLES COLLIDE
  - A LIGHT WAVE HITS A DETECTOR

- TWO EVENTS THAT TAKE PLACE AT DIFFERENT POSITIONS BUT AT THE SAME TIME ARE SAID TO BE SIMULTANEOUS.

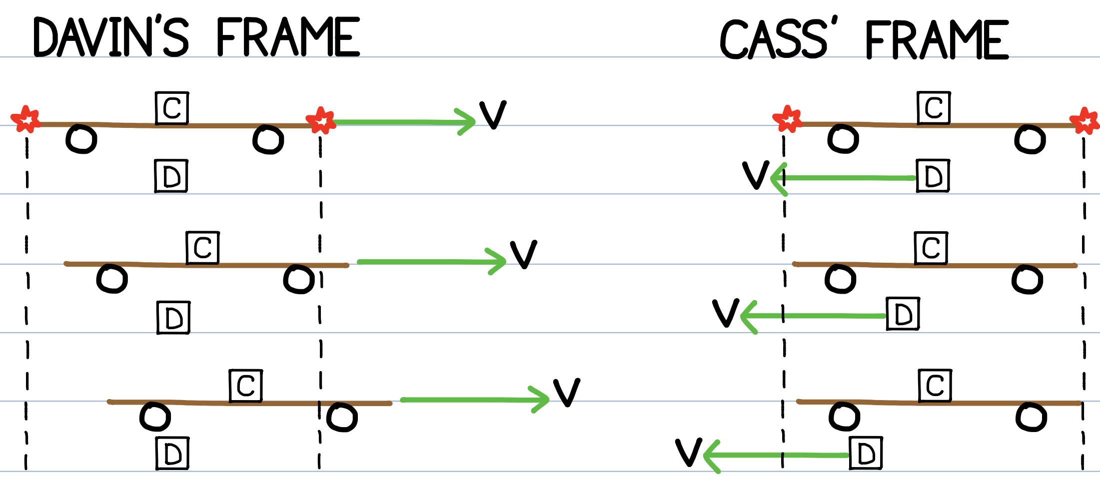
CONSIDER THIS SCENARIO:



- FIRECRACKERS ARE ATTACHED TO THE ENDS OF CASS' RAILROAD CAR.
- THE FIRECRACKERS EXPLODE AND LEAVE SKID MARKS ON THE GROUND.

- THE FLASHES OF LIGHT ARE RECEIVED BY A LIGHT DETECTOR WITH A SIGNAL LIGHT THAT TURNS GREEN IF BOTH SIDES RECEIVE A SIGNAL AT THE SAME TIME OR RED IF EITHER SIDE RECEIVES A SIGNAL BEFORE THE OTHER.
- DAVIN SEES THE FLASHES AT THE SAME TIME AND MEASURES AN EQUAL DISTANCE TO EACH OF THE SKID MARKS. HE CONCLUDES THAT THE TWO FIRECRACKERS EXPLODED SIMULTANEOUSLY.

DOES THE LIGHT TURN GREEN OR RED?



- TWO EVENTS OCCURRING SIMULTANEOUSLY TO ONE OBSERVER ARE NOT NECESSARILY SIMULTANEOUS TO ANOTHER OBSERVER.

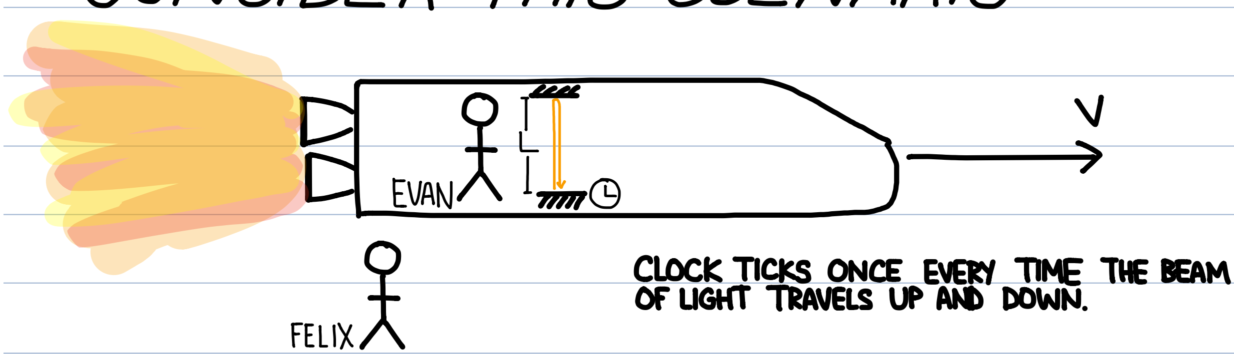
TIME DILATION

- IF TWO EVENTS OCCUR AT THE SAME LOCATION, THEN THE TIME INTERVAL MEASURED BETWEEN THEM IN THAT FRAME OF REFERENCE IS CALLED THE PROPER TIME. A CLOCK AT REST AND PRESENT AT BOTH EVENTS MEASURES PROPER TIME
- IN ANY OTHER INERTIAL FRAME OF REFERENCE, THE TWO EVENTS OCCUR AT DIFFERENT LOCATIONS AND THE TIME INTERVAL IS DILATED (STRETCHED OUT). CLOCKS MOVING RELATIVE AN OBSERVER RUN SLOWER.

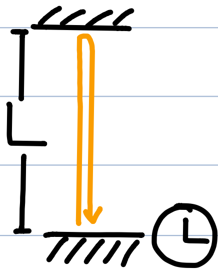
$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

t : DILATED TIME INTERVAL
t_0 : PROPER TIME
v : RELATIVE SPEED
c : SPEED OF LIGHT
$(3.00 \times 10^8 \frac{m}{s})$

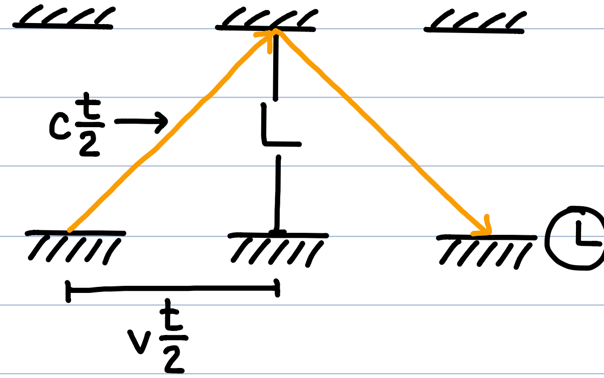
CONSIDER THIS SCENARIO:



EVAN'S FRAME



FELIX'S FRAME



CLOCK IS AT REST

$$t_0 = \frac{2L}{c}$$

$$L = c\frac{t_0}{2}$$

$$L^2 + \left(v\frac{t}{2}\right)^2 = \left(c\frac{t}{2}\right)^2$$

$$\left(c\frac{t_0}{2}\right)^2 + \left(v\frac{t}{2}\right)^2 = \left(c\frac{t}{2}\right)^2$$

$$\frac{c^2 t_0^2}{4} + \frac{v^2 t^2}{4} = \frac{c^2 t^2}{4}$$

$$c^2 t_0^2 + v^2 t^2 = c^2 t^2$$

$$t^2 (c^2 - v^2) = c^2 t_0^2$$

$$t^2 = \left(\frac{c^2}{c^2 - v^2}\right) t_0^2$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

EXAMPLE

A STATIONARY NEUTRON HAS A HALF-LIFE OF 800 s. IF THE NEUTRON MOVES AT A SPEED OF 0.97 c RELATIVE TO AN OBSERVER, WHAT WOULD THE OBSERVER MEASURE THE HALF-LIFE OF THE NEUTRON TO BE?

EXAMPLE

ALPHA CENTAURI IS THE CLOSEST STAR TO THE SUN AT A DISTANCE OF 4.5 LIGHT-YEARS. IF A SPACESHIP TRAVELS AT $\frac{2}{3}c$,

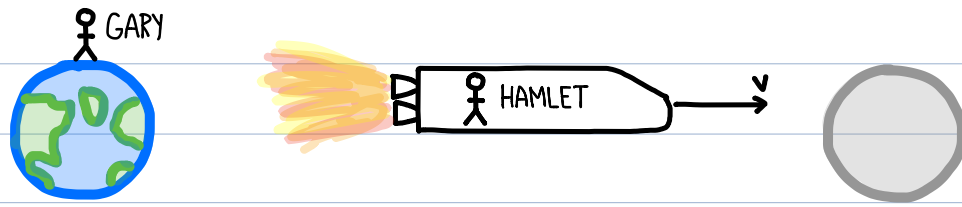
a) HOW LONG IS THE JOURNEY FOR AN OBSERVER ON EARTH?

b) HOW LONG IS THE JOURNEY FOR AN OBSERVER ON THE SPACESHIP?

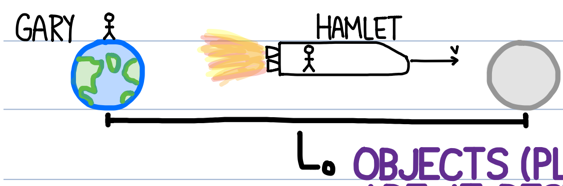
ONE LIGHT-YEAR IS THE <u>DISTANCE</u> LIGHT TRAVELS IN ONE YEAR ($d = vt = 1\text{ year} \times c$)

LENGTH CONTRACTION

CONSIDER THIS SCENARIO:

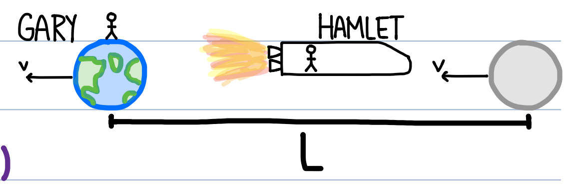


GARY'S FRAME



OBJECTS (PLANETS) ARE AT REST

HAMLET'S FRAME



$$v = \frac{L_0}{t} \qquad v = \frac{L}{t_0}$$

$$\frac{L_0}{t} = \frac{L}{t_0}$$

$$L = \frac{t_0}{t} L$$

$$L = \frac{t_0}{\frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}} L_0$$

$$L = \sqrt{1-\frac{v^2}{c^2}}\, L_0$$

- THE LENGTH OF AN OBJECT (OR DISTANCE BETWEEN TWO OBJECTS) DECREASES IN THE DIRECTION OF MOTION FOR AN OBSERVER MOVING WITH RESPECT TO THE OBJECT(S).

$$L = \sqrt{1-\frac{v^2}{c^2}}\, L_0$$

L: CONTRACTED LENGTH (OBJECT IS MOVING)

L_0: PROPER LENGTH (LENGTH WHEN OBJECT IS AT REST)

V: RELATIVE SPEED

C: SPEED OF LIGHT ($3.00 \times 10^8 \frac{m}{s}$)

EXAMPLE

A SPACESHIP IS TRAVELLING AWAY FROM EARTH AT A SPEED OF 0.8c. AN ASTRONAUT IN THE SPACESHIP MEASURES HIS SHIP TO BE 100 m LONG. WHAT DOES AN OBSERVER ON EARTH MEASURE THE LENGTH TO BE?

MASS INCREASE

- THE MASS OF AN OBJECT INCREASES FOR AN OBSERVER MOVING WITH RESPECT TO AN OBJECT.

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

m: APPARENT MASS (OBJECT IS MOVING)

m_0: REST MASS (MASS WHEN OBJECT IS AT REST)

v: RELATIVE SPEED

c: SPEED OF LIGHT ($3.00 \times 10^8 \frac{m}{s}$)

EXAMPLE

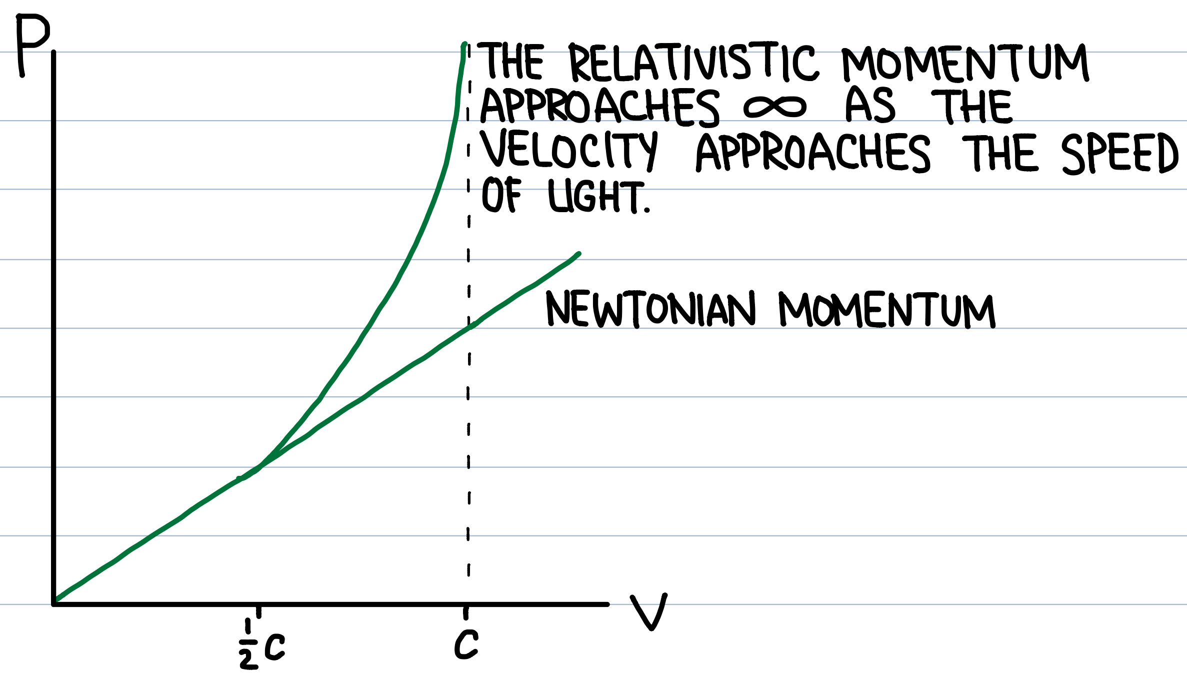
AN OBSERVER MEASURES THE MASS OF A TRAIN MOVING AT A SPEED OF 0.6c WITH RESPECT TO HIMSELF. IF THE MASS HE MEASURED WAS 24 000 kg, WHAT WOULD A A PASSENGER ON THE TRAIN MEASURE ITS MASS TO BE?

RELATIVISTIC MOMENTUM

- MOMENTUM IS CONSERVED IF IT IS REDEFINED AS

$$P = \frac{m_0 V}{\sqrt{1 - \frac{V^2}{C^2}}}$$

P: MOMENTUM, $kg\frac{m}{s}$
m_0: REST MASS, kg
(MASS WHEN OBJECT IS AT REST)
V: RELATIVE SPEED, $\frac{m}{s}$
C: SPEED OF LIGHT
($3.00 \times 10^8 \frac{m}{s}$)



- **THE COSMIC SPEED LIMIT**: A MATERIAL PARTICLE CAN NEVER REACH THE SPEED OF LIGHT BECAUSE THE PARTICLE'S MOMENTUM BECOMES INFINITELY LARGE AS THE SPEED APPROACHES c. THE AMOUNT OF EFFORT REQUIRED FOR EACH ADDITIONAL INCREMENT IN VELOCITY BECOMES LARGER AND LARGER UNTIL NO AMOUNT OF EFFORT CAN RAISE THE VELOCITY ANY HIGHER.

MASS-ENERGY EQUIVALENCE

- THE WORK-ENERGY THEOREM IS STILL VALID IN RELATIVITY IF THERE IS AN ENERGY ASSOCIATED WITH MASS ITSELF.

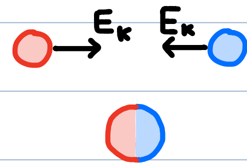
$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} = E_0 + E_k$$

$$E_0 = m_0 c^2$$

E : TOTAL ENERGY, J
E_0 : REST ENERGY, J
E_k : KINETIC ENERGY, J
m_0 : REST MASS, kg
m : APPARENT MASS, kg
v : RELATIVE SPEED, $\frac{m}{s}$
c : SPEED OF LIGHT ($3.00 \times 10^8 \frac{m}{s}$)

EXAMPLE

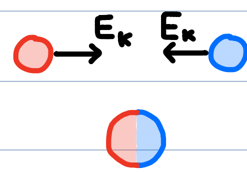
- AN INELASTIC COLLISION BETWEEN TWO CLAY BALLS



$E_i = 2m_0c^2 + 2E_K$

$E_f = 2m_0c^2$?

- IN NEWTONIAN MECHANICS, WE WOULD SAY THAT THE INITIAL KINETIC ENERGY $2E_K$ IS DISSIPATED BY BEING TRANSFORMED INTO AN EQUAL AMOUNT OF THERMAL ENERGY, RAISING THE TEMPERATURE OF THE BALL.
- $E = E_0 + E_K$ DOES NOT SAY ANYTHING ABOUT THERMAL ENERGY.
- TOTAL ENERGY CAN BE CONSERVED ONLY IF KINETIC ENERGY IS TRANSFORMED INTO AN EQUIVALENT AMOUNT OF MASS



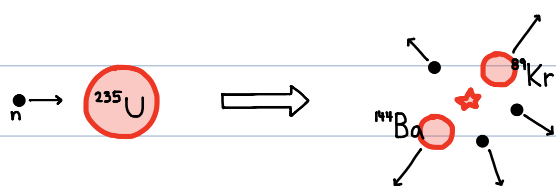
$E_i = 2m_0c^2 + 2E_K$

~~$E_f = 2m_0c^2$?~~ $E_f = M_0c^2$

$M_0 > 2m_0$

- NUCLER REACTIONS

FISSION:



THE MASS OF THE REACTANTS IS 3.07×10^{-28} kg MORE THAN THE MASS OF THE PRODUCTS.

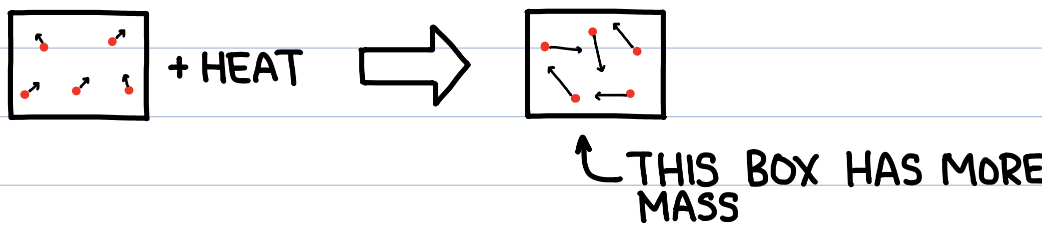
3.07×10^{-28} kg OF MASS HAS BEEN CONVERTED INTO KINETIC ENERGY.

FUSION:

$${}^{2}_{1}H + {}^{3}_{1}H \longrightarrow {}^{4}_{2}He + {}^{1}_{0}n + \underbrace{17.6\ \text{MeV}}_{\text{ENERGY}}$$

- THE ENERGY RELEASED IN A FUSION REACTION IS FROM THE LOSS OF MASS.

- HEATING UP A BOX OF GAS



- MASS IS NOT CONSERVED!

RELATIVISTIC ADDITION OF VELOCITIES

- AT RELATIVISTIC SPEEDS, CLASSICAL VECTOR ADDITION CAN RESULT IN PROJECTILES MOVING FASTER THAN THE SPEED OF LIGHT.

- RELATIVISTIC ADDITION OF VELOCITIES:

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

v: RELATIVE VELOCITY BETWEEN THE TWO FRAMES OF REFERENCE

u': VELOCITY OF PROJECTILE IN THE FRAME MOVING AT v

u: VELOCITY OF THE PROJECTILE IN THE FRAME AT REST

EXAMPLE

AARON IS TRAVELLING TOWARDS BRIAN AT A SPEED OF 0.6c. AARON THROWS A BALL TOWARDS BRIAN AT A SPEED OF 0.6c (IN HIS FRAME OF REFERENCE). AT WHAT SPEED DOES BRIAN SEE THE BALL APPROACH HIM?